One-Dimensional Parabolic Partial Differential Equations

(II) Boundary Value problems

Example 1: The Black-Scholes equation is famous for the use in modelling derivative prices in financial mathematics. In transformed and dimensionless form [3], using parameter values from [2], we have

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + (k-1)\frac{\partial u}{\partial x} - ku, \qquad a \le x \le b, \qquad t_0 \le t \le t_f,$$

where $k = 2r/\sigma^2$, $r = 0.065$, $\sigma = 0.8$, $a = \log(7/5)$, $t_0 = 0$, $t_f = 5$, with initial condition

$$u(x,0) = \max(\exp x - 1, 0)$$

and boundary conditions

$$u(a,t) = 0, \qquad u(b,t) = \frac{7 - 5\exp(-kt)}{5}.$$

This problem can be solved by using Matlab function "pdepe", see [1] for the details.

References

[1] D. J. Higham and N. J. Higham. *Matlab Guide.* SIAM, 2000.

[2] S. N. Neftci. *An Introduction to the Mathematics of Financial Derivatives.* Academic Press, 1996.

[3] P. Wilmott, S. Howison, and J. Dewynne. *The Mathematics of Financial Derivatives: A Student Introduction.* Cambridge University Press, 1995.