91學年度第1學期

課程編號: 221 U1310 學分: 3
科目名稱: 數值偏微分方程式一
課程網站: http://www.math.ntu.edu.tw/~shyue/myclass/npde02

Homework ♯ 4

Assign: 11/20/2002 Due: 12/4/2002

• Include your computer program(s), when turning the homework set

1. Consider the semi-discretized scheme

$$\frac{du_j}{dt} = \frac{1}{(\Delta x)^2}\left(\frac{11}{12}u_{j-1} - \frac{5}{3}u_j + \frac{1}{2}u_{j+1} + \frac{1}{3}u_{j+2} - \frac{1}{12}u_{j+3}\right), \qquad j \in \mathbb{Z},$$

for the diffusion equation $u_t = u_{xx}$.

a) Determine the order of the accuracy of the scheme.

b) Is the scheme stable ?

2. Consider the finite-difference scheme

$$u_j^{n+1} = \frac{1}{2}\left(2 - 5\mu + 6\mu^2\right)u_j^n + \frac{2}{3}\mu\left(2 - 3\mu\right)\left(u_{j-1}^n + u_{j+1}^n\right) - \frac{1}{12}\mu\left(1 - 6\mu\right)\left(u_{j-2}^n + u_{j+2}^n\right), \qquad j \in \mathbb{Z}.$$

Use the Fourier technique to show that the scheme is stable only when $0 \le \mu \le 2/3$.

3. Consider the following system of reaction-diffusion equations

$$\frac{\partial u}{\partial t} = \alpha\frac{\partial^2 u}{\partial x^2} + A + u^2 v - (B+1)u$$
$$\frac{\partial v}{\partial t} = \alpha\frac{\partial^2 v}{\partial x^2} + Bu - u^2 v$$

with $0 \le x \le 1$, $A = 1$, $B = 2$, $\alpha = 1/50$, and boundary conditions

$$u(0,t) = u(1,t) = 1, \qquad v(0,t) = v(1,t) = 3,$$
$$u(x,0) = 1 + \sin(2\pi x), \qquad v(x,0) = 3.$$

a) Solve this problem by using a semi-discretization of the method with a uniform spatial mesh size h (say $h = 1/50$), and an *explicit* solver for the resulting system of ODEs. Draw the surface plot of the solutions $u(x,t)$ and $v(x,t)$ for $0 \le t \le 10$.

b) Do the same experiment as in a), but with an *implicit* solver for the resulting system of ODEs.