

91學年度第1學期

課程編號: 221 U1310

學分: 3

科目名稱: 數值偏微分方程式一

課程網站: <http://www.math.ntu.edu.tw/~shyue/myclass/npde02>

Homework # 4

Assign: 11/20/2002

Due: 12/4/2002

- Include your computer program(s), when turning the homework set

1. Consider the semi-discretized scheme

$$\frac{du_j}{dt} = \frac{1}{(\Delta x)^2} \left(\frac{11}{12}u_{j-1} - \frac{5}{3}u_j + \frac{1}{2}u_{j+1} + \frac{1}{3}u_{j+2} - \frac{1}{12}u_{j+3} \right), \quad j \in \mathbb{Z},$$

for the diffusion equation $u_t = u_{xx}$.

a) Determine the order of the accuracy of the scheme.

b) Is the scheme stable ?

2. Consider the finite-difference scheme

$$u_j^{n+1} = \frac{1}{2} (2 - 5\mu + 6\mu^2) u_j^n + \frac{2}{3}\mu (2 - 3\mu) (u_{j-1}^n + u_{j+1}^n) - \frac{1}{12}\mu (1 - 6\mu) (u_{j-2}^n + u_{j+2}^n), \quad j \in \mathbb{Z}.$$

Use the Fourier technique to show that the scheme is stable only when $0 \leq \mu \leq 2/3$.

3. Consider the following system of reaction-diffusion equations

$$\begin{aligned} \frac{\partial u}{\partial t} &= \alpha \frac{\partial^2 u}{\partial x^2} + A + u^2 v - (B + 1)u \\ \frac{\partial v}{\partial t} &= \alpha \frac{\partial^2 v}{\partial x^2} + Bu - u^2 v \end{aligned}$$

with $0 \leq x \leq 1$, $A = 1$, $B = 2$, $\alpha = 1/50$, and boundary conditions

$$\begin{aligned} u(0, t) &= u(1, t) = 1, & v(0, t) &= v(1, t) = 3, \\ u(x, 0) &= 1 + \sin(2\pi x), & v(x, 0) &= 3. \end{aligned}$$

- a) Solve this problem by using a semi-discretization of the method with a uniform spatial mesh size h (say $h = 1/50$), and an *explicit* solver for the resulting system of ODEs. Draw the surface plot of the solutions $u(x, t)$ and $v(x, t)$ for $0 \leq t \leq 10$.
- b) Do the same experiment as in a), but with an *implicit* solver for the resulting system of ODEs.