91學年度第1學期

課程編號: 221 U1310　　學分: 3
科目名稱: 數值偏微分方程式一

Homework ♯ 1

Due 10/09/2002

- Include your computer program(s), when turning the homework set

1. In aerodynamics, one of the most important early applications (the boundary layer theory) is Blasius's equation (see [2] for a brief introduction of the theory),

$$2f^{'''} + ff^{''} = 0 \tag{1}$$

which gives the incompressible velocity profile for a flat plate. For this problem, three initial conditions are known as

$$f(0) = 0, \quad f'(0) = 0, \quad \text{and } f^{''}(0) = 0.332,$$

and the independent variable is usually denoted by η.

a) Solve this problem numerically up to $\eta = 6$, and then plot η versus $f^{'}(\eta)$. You may use your-own ODE solver, the free public ODE solvers from Netlib (see http://www.math.ntu.edu.tw/ shyue/myclass/npde02), or any of the ODE routines provided in Matlab.

b) The momentum thickness for an incompressible flow is proportional to

$$\theta(u) = \int_0^u f^{'}(1 - f^{'})d\eta.$$

Find $\theta(5.0)$.

2. In hydrodynamics, it is known that the Rayleigh-Plesset equation of the form,

$$R\frac{d^2R}{dt^2}+\frac{3}{2}\left(\frac{dR}{dt}\right)^2=\frac{1}{\rho_L}\left[p_G(R,t)-p_e(t)\right]+\frac{R}{\rho_L c_L}\frac{d}{dt}\left[p_G(R,t)-p_e(t)\right]-\frac{4\nu_L}{R}\frac{dR}{dt}, \qquad (2)$$

provides the leading-order description of the breathing motion of a gas-filled bubble driven by a sound field, see [1] or [3] for the details. Here $R(t)$ is the radius of the bubble, p_e is the acoustic drive at the bubble, p_G is the pressure of the gas inside the bubble, and ρ_L, c_L , and ν_L are the density, speed of sound, and kinematic viscosity of the fluid, respectively.

Concerning p_e and p_G, in this problem set, we are mainly interested in the following conditions:

$$p_e(t)=p_0+\alpha\sin(\omega t), \qquad p_G(R,t)=p_0\left(\frac{R_0^3-\beta^3}{R^3-\beta^3}\right)^\gamma$$

which are closely related to many important instances in the sonoluminescence experiments. Here p_0 is the ambient atmospheric pressure, α and ω are the amplitude and frequency of the acoustic drive, respectively, γ is the ratio of specific heats, and β is the radius of the van der Waals hard core of the gas.

a) Now consider the parameter values: $p_0 = 1$ atm (*i.e.*, $\approx 1.013\times 10^5$Pa), $\rho_L = 10^3$ kg/m^3, $c_L = 1481$ m/s, $\nu_L = 7\times 10^{-5}$m^2/s, $\gamma = 1.4$, and $\beta = 0$, solve (2) with $\alpha = -1.275$ atm, $\omega/(2\pi) = 26.5$ kHz, and the initial condition $R(0) = R_0 = 4.5\mu$m up to time $t \geq 25\mu$ s.

b) Using the same parameters as in a), investigate the solution behavior of (2) with different choices of R_0, α, and ω.

c) Would the solution behavior computed in b) vary significantly when (i) $\nu_L = 0$, (ii) $\beta = R_0/8.54$?

References

[1] S. Hilgenfeldt, M. P. Brenner, S. Grossmann, and D. Lohse. Analysis of Rayleigh-Plesset dynamics for sonoluminescing bubbles. *J. Fluid Mech.*, 365:171–204, 1998.

[2] R. L. Panton. *Incompressible Flow.* John Wiley & Sons, Inc., 1984.

[3] C. C. Wu and P. H. Roberts. A model of sonoluminescence. *Proc. R. Soc. Lon. A*, 445:323–349, 1994.