92學年微甲統一教學期末考 (下學期)

**1.(15%)** 在球坐標$(\rho,\phi,\theta)$ 之下, 球面$\rho=a$與圓錐面$\phi=\frac{\pi}{6}$所圍成的領域, 求其質心的坐標$(\overline{x},\overline{y},\overline{z})$ 。

**2.(10%)** Find the area of the part of the sphere $x^2+y^2+z^2=a^2$ that lies within the cylinder $x^2+y^2=ax$ and above the $xy$-plane 。

**3.(10%)** 求積分$\iint_D \sin(9x^2+4y^2)dA$, 其中 $D$ 爲$9x^2+4y^2\leq 1$之領域 。

**4.(10%)** 設曲線$C$ : $\vec{r}(t)=\cos t\ \vec{i}+\sin t\ \vec{j}+t\ \vec{k}$, $0\leq t\leq 6\pi$, 其密度爲$\rho(x,y,z)=1+xz$, 求此曲線的質量 。

**5.(10%)** 令向量場 $\vec{F}=(e^x\sin xy+ye^x\cos xy)\vec{i}+(e^xy\cos xy+z)\vec{j}+(ze^z+y)\vec{k}$, $C$爲由$(0,2,1)$到$(1,\frac{\pi}{2},2)$之任意平滑曲線

[a] 求$\vec{F}$的位能函數 $f$ (potential function), 即$\nabla f=F$ 。

[b] 求$\int_C \vec{F}\cdot\vec{T}ds$之值 。

**6.(10%)** Let $\vec{a}$ be a constant vector. $\vec{r}=x\ \vec{i}+y\ \vec{j}+z\ \vec{k}$ is the position vector of $(x,y,z)$, $r=|\vec{r}|$ 。
Define the vector field $\vec{V}(x,y,z)=r^n\vec{a}\times\vec{r}$,where $n$ is a positive integer. Find div$(\vec{V})$ and curl$(\vec{V})$ 。

**7.(10%)** Find the work done by the force $\vec{F}(x,y)=(x(x+y))\vec{i}+(xy^2)\vec{j}$ in moving a particle from the origin along the $x$-axis to $(1,0)$, then along the line segment to $(0,1)$, and then back to the origin along the $y$-axis。

**8.(15%)** Let $(a,b,c)$ be a fixed point on the sphere $S:x^2+y^2+z^2=R^2$ ($R>0$ is the positive radius). The mass density $\rho(x,y,z)$ at $(x,y,z)$ on $S$ is the distance from $(x,y,z)$ to $(a,b,c)$(i.e. $\rho(x,y,z)=\sqrt{(x-a)^2+(y-b)^2+(z-c)^2}$). Find the total mass of $S$ 。

**9.(15%)** Find the outward flux of the vector field
$\vec{V}(x,y,z) = ((x/r^3) + y + z)\ \vec{i} + ((y/r^3) + x + z)\ \vec{j} + ((z/r^3) + x + y)\ \vec{k}$
across the boundary of the ellipsoid region $D : 10x^2 + 11y^2 + 12z^2 \leq 13$,
where $r = \sqrt{x^2 + y^2 + z^2}$ 。

**10.(15%)** Define the vector field on the plane by

$$\vec{V}(x,y) = \frac{-y\ \vec{i} + (x^2 + y^2 - x)\ \vec{j}}{(x-1)^2 + y^2}$$

Prove that

[a](5%) $\mathrm{curl}(\vec{V}) = 0$ 。

[b](10%) Compute the line integral $\oint_\Gamma V \cdot d\vec{r}$, where $\Gamma$ is a simple closed curve without passing $(1,0)$ 。