93學年上學期微甲一組期末考
Final Examination of Calculus, Jan. 9, 2004

Each question is 10 points.

1. Evaluate the following limits.

   (a) $\lim_{x\to 0^+}\left(\frac{1}{x^2}-\frac{1}{x\tan x}\right)$.

   (b) $\lim_{x\to 0}\left(\frac{\cos x}{\cos 3x}\right)^{1/x^2}$.

2. Find $\int x\left(\sec^{-1} x\right)^2\ dx$.

3. Find $\int \frac{dx}{e^{\frac{x}{2}}+e^{\frac{x}{3}}+e^{\frac{x}{6}}}$.

4. Find the volume of a solid torus which is obtained by rotating the circle $(x-R)^2+y^2=r^2$, $R>r>0$, about the y-axis.

5. If the infinite curve $y=e^{-x}$, $x\geq 0$, is rotated about the x-axis, find the area of the resulting surface.

6. Determine whether the series $\sum_{n=1}^{\infty}(-1)^{[\sqrt{n}]}\frac{(n!)^k}{(kn)!}$ is absolutely convergent, conditionally convergent, or divergent, where $[\sqrt{n}]$ is the largest integer no greater than $\sqrt{n}$, and k is a positive integer.

7. Determine whether the series $\sum_{n=1}^{\infty}(-1)^{n-1}\left(\sqrt[n]{n}-1\right)$ is absolutely convergent, conditionally convergent, or divergent.

8. Compute $f^{(13)}(0)$ when $f(x)=e^{x^5e^x}$.

9. Find the Taylor series of $\ln\left(2+2x-x^2\right)$ centered at 1, and determine the interval of convergence of this Taylor series.

10. Use power series to approximate $\int_{\frac{1}{2}}^{1}\cos x^2\ dx$ correct up to three decimal places. Carefully justify that your answer is indeed correct up to three decimal places.(請以小數形式寫下你的近似值與誤差)