1. (14%) Find the maximum and the minimum of $f(x,y) = xy$ subject to the constraint $x^2+xy+y^2 = 1$.

2. (12%) Let $f(x,y) = x^3 - 3\lambda xy + y^3$, where $\lambda \neq 0$ is a real number. Find all the critical points of f. Determine which give rise to local maxima, local minima, saddle points. (Note: It depends on the value of λ.)

3. (12%) Evaluate $\iint\limits_{\Omega} \frac{x^2}{x^2+y^2} dA$, where Ω is the region $1 \leq x^2 + y^2 \leq 2$.

4. (12%) Evaluate $\iint\limits_{\Omega} (y-x)(2x+y) dA$, where Ω is the region enclosed by $y-x = 1$, $y-x = 2$, $2x+y = 0$, and $2x + y = 2$.

5. (12%) Let $f(x,y)$ be differentiable. Suppose that $\frac{\partial f}{\partial x}(2,-2) = \sqrt{2}$, and $\frac{\partial f}{\partial y}(2,-2) = \sqrt{5}$. Let $x = u - v$ and $y = v - u$. Find the value of $\frac{\partial f}{\partial u} + \frac{\partial f}{\partial v}$ at $(u,v) = (1,-1)$.

6. (14%) Let $f(x,y) = e^x \cos y + a \sin y$, where a is a constant.
   (a) (7%) Find an equation of the tangent plane to the level curve $f(x,y) = -1$ at the point $(0,\pi)$. (The equation may contain a.)
   (b) (7%) Suppose the maximum of the directional derivative $\frac{\partial f}{\partial \vec{u}}(0,0)$ occur at $\vec{u} = \left(\frac{3}{5}, \frac{4}{5}\right)$, find the value of a.

7. (12%) Evaluate $\iint\limits_{\Omega} \frac{3x^2}{(x^3+y^2)^2} dA$, where $\Omega = [0,1] \times [1,3]$.

8. (12%) Evaluate $\iint\limits_{R} \frac{xe^y}{y} dA$, where $R: 0 \leq x \leq 1$ and $x^2 \leq y \leq x$.