1. (12%) Solve the differential equation $y' = y^2(1-y)$ satisfying the initial condition $y = 2$ when $t = \ln 2$.
2. (12%) Solve the differential equation $y' + y = \sin t$.
3. (15%) Let the service time of attendants A and B be X and Y. Suppose that they are independently exponential distributed with mean a and b, repectively.
   (a) Find the probability density functions X and Y.
   (b) Find the joint probability density function.
   (c) Find $P(X \geq Y)$.
4. (12%) On average, five public security incidents occurred every ten years. Assume it follows the Poisson distribution. What is the probability that there is no public security incident within 4 years? What is the probability that there are two incidents within 4 years?
5. (12%) Let $f_X(t) = \lambda e^{-\lambda t}$, where $t \geq 0$ and λ is a positive constant. Find the probability density function of the random variable $\sqrt{X}$.
6. (12%) There are 1000 attendants in one exam. 80 people will be admitted. Full score is 100. Known average 65, variance 25. Use Chebyshev's inequality to estimate the lowest possible score for admission.
7. (10%) Consider the improper integral $\int_1^\infty \frac{dx}{x^p}$, where $p > 0$. For which value(s) of p, does the integral converge?
8. (15%) X and Y are random variables and have values $\{1, 2\}$ and $\{1, 2, 3\}$, respectively. The joint probability is the following

$$P(X=1, Y=1) = \frac{2}{7}, \quad P(X=1, Y=2) = \frac{3}{14}, \quad P(X=1, Y=3) = \frac{1}{7}$$
$$P(X=2, Y=1) = \frac{3}{14}, \quad P(X=2, Y=2) = \frac{1}{14}, \quad P(X=2, Y=3) = \frac{1}{14}.$$

   (a) (5%) Find $\mathrm{E}(X)$ and $\mathrm{E}(Y)$,
   (b) (5%) Find $\mathrm{Var}(X)$ and $\mathrm{Var}(Y)$,
   (c) (5%) Determine whether X and Y are independent.