Section 4.9 Antiderivatives

4. Find an antiderivative of the function. (a) $g(t) = 1/t$ (b) $r(\theta) = \sec^2\theta$

Solution:

(a) $g(t) = 1/t \quad \Rightarrow \quad G(t) = \ln|t|$ is an antiderivative.

(b) $r(\theta) = \sec^2\theta \quad \Rightarrow \quad R(\theta) = \tan\theta$ is an antiderivative.

12. Find the most general antiderivative of the function. (Check your answer by differentiation.)

$$h(z) = 3z^{0.8} + z^{-2.5}$$

Solution:

$$h(z) = 3z^{0.8} + z^{-2.5} \quad \Rightarrow \quad H(z) = 3\,\frac{z^{1.8}}{1.8} + \frac{z^{-1.5}}{-1.5} = \frac{5}{3}\,z^{1.8} - \frac{2}{3}\,z^{-1.5} + C$$

24. Find the most general antiderivative of the function. (Check your answer by differentiation.)

$$g(v) = 2\cos v - \frac{3}{\sqrt{1-v^2}}$$

Solution:

$$g(v) = 2\cos v - \frac{3}{\sqrt{1-v^2}} \quad \Rightarrow \quad G(v) = 2\sin v - 3\sin^{-1} v + C$$

52. If $f''(t) = \sqrt[3]{t} - \cos t$, $f(0) = 2$, $f(1) = 2$. Find f.

Solution:

$f''(t) = \sqrt[3]{t} - \cos t = t^{1/3} - \cos t \quad \Rightarrow \quad f'(t) = \frac{3}{4}t^{4/3} - \sin t + C \quad \Rightarrow \quad f(t) = \frac{9}{28}t^{7/3} + \cos t + Ct + D.$

$f(0) = 0 + 1 + 0 + D$ and $f(0) = 2 \quad \Rightarrow \quad D = 1$, so $f(t) = \frac{9}{28}t^{7/3} + \cos t + Ct + 1.$ $f(1) = \frac{9}{28} + \cos 1 + C + 1$ and

$f(1) = 2 \quad \Rightarrow \quad C = 2 - \frac{9}{28} - \cos 1 - 1 = \frac{19}{28} - \cos 1$, so $f(t) = \frac{9}{28}t^{7/3} + \cos t + \left(\frac{19}{28} - \cos 1\right) t + 1.$