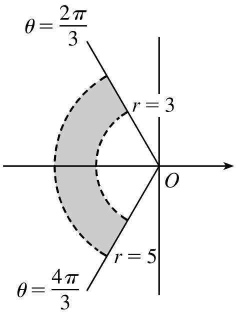
Section 10.3 Polar Coordinates

10. Sketch the region in the plane consisting of points whose polar coordinates satisfy the given conditions.

$$3 < r < 5, \quad 2\pi/3 \le \theta \le 4\pi/3$$

Solution:

$3 < r < 5,\ 2\pi/3 \le \theta \le 4\pi/3$



18. Identify the curve by finding a Cartesian equation for the curve.

$$\theta = \pi/3$$

Solution:

$\theta = \dfrac{\pi}{3} \quad \Rightarrow \quad \tan\theta = \tan\dfrac{\pi}{3} \quad \Rightarrow \quad \dfrac{y}{x} = \sqrt{3} \quad \Leftrightarrow \quad y = \sqrt{3}\,x$, a line through the origin.

25. Find a polar equation for the curve represented by the given Cartesian equation.

$$x^2 + y^2 = 4y$$

Solution:

$x^2 + y^2 = 4y \quad \Rightarrow \quad r^2 = 4r\sin\theta \quad \Rightarrow \quad r^2 - 4r\sin\theta = 0 \quad \Rightarrow \quad r(r - 4\sin\theta) = 0 \quad \Rightarrow \quad r = 0$ or $r = 4\sin\theta$.

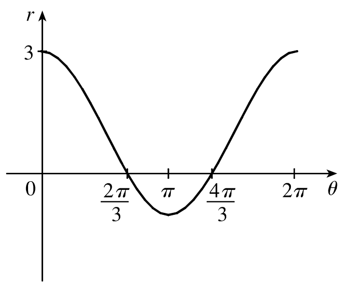
$r = 0$ is included in $r = 4\sin\theta$ when $\theta = 0$, so the curve is represented by the single equation $r = 4\sin\theta$.

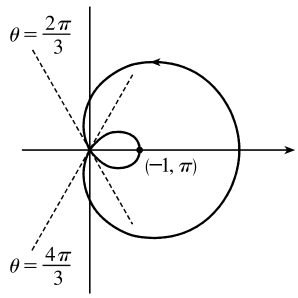
36. Sketch the curve with the given polar equation by first sketching the graph of r as a function of θ in Cartesian coordinates.

$$r = 1 + 2\cos\theta$$

Solution:

$r = 1 + 2\cos\theta$

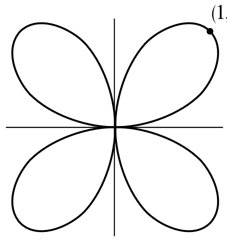




54. Sketch the curve $(x^2+y^2)^3 = 4x^2y^2$.

Solution:

The equation is $(x^2+y^2)^3 = 4x^2y^2$, but using polar coordinates we know that $x^2+y^2 = r^2$ and $x = r\cos\theta$ and $y = r\sin\theta$. Substituting into the given equation: $r^6 = 4r^2\cos^2\theta\, r^2\sin^2\theta \quad\Rightarrow\quad r^2 = 4\cos^2\theta\,\sin^2\theta \quad\Rightarrow$ $r = \pm 2\cos\theta\,\sin\theta = \pm\sin 2\theta$. $r = \pm\sin 2\theta$ is sketched at right.



58. Show that the curves $r = a\sin\theta$ and $r = a\cos\theta$ intersect at right angles.

Solution:

These curves are circles which intersect at the origin and at $\left(\frac{1}{\sqrt{2}}a, \frac{\pi}{4}\right)$. At the origin, the first circle has a horizontal tangent and the second a vertical one, so the tangents are perpendicular here. For the first circle [$r = a\sin\theta$], $dy/d\theta = a\cos\theta\,\sin\theta + a\sin\theta\,\cos\theta = a\sin 2\theta = a$ at $\theta = \frac{\pi}{4}$ and $dx/d\theta = a\cos^2\theta - a\sin^2\theta = a\cos 2\theta = 0$ at $\theta = \frac{\pi}{4}$, so the tangent here is vertical. Similarly, for the second circle [$r = a\cos\theta$], $dy/d\theta = a\cos 2\theta = 0$ and $dx/d\theta = -a\sin 2\theta = -a$ at $\theta = \frac{\pi}{4}$, so the tangent is horizontal, and again the tangents are perpendicular.