This note is to answer several questions concerning about nonmeasurable sets. First of all, I would like to give a construction in Dudley's book. Let α be an irrational number, say $\alpha = \sqrt{2}$. Consider two subgroups of $\mathbb{R}$, $G = \{m + n\alpha : m, n \in \mathbb{Z}\}$ and $H = \{2m + n\alpha : m, n \in \mathbb{Z}\}$. It is clear that G is the disjoint union of H and $H + 1$. One can easily show that G, H, and $H+1$ are all dense subsets of $\mathbb{R}$. Let $y \in \mathbb{R}$, the cosets $G+y$ are either disjoint or identical. Using the axiom of choice, let C be the sets consisting exactly one element from each coset $G + y$. Let $X = C + H$, then $X^c = C + H + 1$. Simple computation shows that $(X - X) \cap (H + 1) = \emptyset$. Since $H + 1$ is dense, $X - X$ can not contain any interval. Using a previous result, X can not contain any measurable set with positive measure. Thus let $E = X \cap I$, then any measurable subset of E is of measure zero. So let $F \subset I$ be any measurable set containing $I \setminus E$, then $I \setminus F \subset E$ and $I \setminus F$ is measurable. Hence $m(I \setminus F) = 0$, i.e., $m(F) = 1$. Therefore, we have $m^*(I \setminus E) = 1$. Similarly, $I \setminus E = X^c \cap I$ and $(X^c - X^c) \cap H + 1 = \emptyset$. X^c can not contain any positive measurable set. Thus, we must have $m^*(E) = 1$. Thus E can not be measurable and the outer measure of E is 1. From the proof, we can also see that E and $I \setminus E$ can not contain any measurable set with positive measure. Finally, you can modify the argument to show that given any $x \in (0, 1]$, there exists a nonmeasurable set E of I such that $m^*(E) = x$.