Real Analysis Homework #4

Due 10/20

1. Let $\mathcal{L}$ be the collection of Lebesgue measurable sets of $\mathbb{R}^1$ and λ the Lebesgue measure on $\mathcal{L}$. Show that $\mathcal{L}$ is invariant under translation and dilation, i.e., if $E \in \mathcal{L}$, then $E + s \in \mathcal{L}$ and $rE \in \mathcal{L}$ for any $s, r \in \mathbb{R}$. Also show that $\lambda(E + s) = \lambda(E)$ and $\lambda(rE) = |r|\lambda(E)$.

2. Let E be Lebesgue measurable and $\lambda(E) < \infty$. Show that for any $\varepsilon > 0$ there exists a finite union of disjoint open intervals $I_1, \cdots, I_n$ such that

$$\lambda(E \triangle \bigcup I_n) < \varepsilon.$$

3. Let A the set of numbers in $[0, 1]$ which decimal expansions do not contain digit 5. Show that $\lambda(A) = 0$.

4. Given $0 < \varepsilon < 1$, construct a *dense* subset $E \subset [0, 1]$ such that $\lambda(E) = \varepsilon$.