

Advanced Algebra I

Homework 6

due on Oct.31, 2003

Part A.

- (1) Suppose that there are two representation ρ, ρ' on V, V' respectively. A linear transformation $T : V \rightarrow V'$ is said to be *G-invariant* if it's compatible with representations. That is,

$$T\rho_s(v) = \rho'_s(Tv),$$

for all $v \in V$. Prove that if $T : V \rightarrow V'$ is G -invariant, then the $\ker(T) \subset V$ and $\text{im}(T) \subset V'$ are G -invariant subspaces.

- (2) A class function χ is called an abelian character if $\chi(st) = \chi(s)\chi(t)$. Let $\hat{G}$ be the set of abelian characters. Show that $\hat{G}$ is naturally a group. And prove that if G is a finite abelian group, then $|G| = |\hat{G}|$.
- (3) Let ρ be a representation of G on V . Prove or disprove: If the only G -invariant linear transformation on V are multiplication by scalar, then ρ is irreducible.

Part B.

- (1) Let G be a non-abelian group of order 27 such that maximal order is 3.
- (a) Show that the center of G is a group of order 3.
 - (b) How many conjugacy classes are there in G ?
 - (c) Determine the character table of G .
 - (d) Find an irreducible representation of degree 3.