

Advanced Algebra I

Homework 13

due on Jan. 2, 2004

- (1) Determine the Galois group of the following extension:
 - (a) $x^4 - 5$ over $\mathbb{Q}(\sqrt{5}), \mathbb{Q}(\sqrt{-5})$ and over $\mathbb{Q}$.
 - (b) $(x^3 - 3)(x^2 - 2)$ over $\mathbb{Q}$.
- (2) Find examples of irreducible polynomials of degree 3, 4 respectively with Galois group S_3, S_4 respectively over the field $\mathbb{Z}_5$.
- (3) Let $K \subset E \subset F$ be extensions. Prove or disprove the following:
 - (a) F/K is separable, then F/E and E/K are separable.
 - (b) If F/E and E/K are separable, then F/K is separable.
- (4) Let $\zeta = e^{\frac{2\pi i}{7}} \in \mathbb{C}$ be a primitive 7th root of unity. Show that $\mathbb{Q}(\zeta)/\mathbb{Q}$ is Galois with Galois group $\cong \mathbb{Z}_7^*$. Determine all the intermediate subfields. And solve ζ by radicals.