

Advanced Algebra I

Homework 11

due on Dec. 12, 2003

- (1) Let E, F be finite extension over K and contained in a larger field L .
 - (a) Show that $[EF : K] \leq [E : K][F : K]$.
 - (b) If $[E : K]$ and $[F : K]$ are relatively prime, show that $[EF : K] = [E : K][F : K]$.
- (2) Let F/K be an algebraic extension. Let u, v be elements in F with minimal polynomial $f(x), g(x)$ respectively. If $\deg(f(x))$ and $\deg(g(x))$ are relatively prime, show that $g(x)$ is irreducible in $K(u)[x]$.
- (3) Let $f(x) \in K[x]$ be a polynomial of degree n . Let F be a splitting field of $f(x)$. Show that $[F : K] | n!$.
- (4) If F is algebraic over K and D is a domain such that $K \subset D \subset F$, then D is a field.
- (5) Consider the field $K(x)$ where x is an indeterminate. Let $\phi(x) = \frac{f(x)}{g(x)}$, determine $[K(x) : K(\phi)]$ and verify your answer.