# Statistical Computing

Homework 1: RNG and Writing R functions

Due Date: March 10th, 2004

1. Read p37 to 41 in R-intro.pdf about random number generator and various plots on examining the distribution of a set of data.
2. Read p45 to 48 in R-intro.pdf about "Grouping, loops and conditional execution" and writing your own function.
3. Write a R program (or other programming language) to implement a uniform random number generator using a multiplicative congruential method with $x_{i+1} = 17x_i \bmod m$ and $m = 2^{13} - 1$. Generate 500 numbers for the starting point $x_0 = 100$. (help For the sequence $u_i = x_i/m$:
   (a) Plot a histogram to display the results. (help(hist))
   (b) Calculate the coefficient correlation of the pairs of successive number $u_i$ and $u_{i+1}$. (help(cor)).
   (c) Plot the pairs $(u_i, u_{i+1})$ on a 2D plot. (help(plot))
4. Write a program to find the period of a random number generator for a given seed. Use this program to find the period of the sequence generated by $x_{i+1} = 7x_i \bmod 13$ and $x_0 = 19$. Find the period if a is changed to 3.
5. Read p69-70 and 78-79 in R-intro.pdf about how to add a graph in terms of "Low-level plotting commands" to existing plot which is produced by "High-level plotting commands."
6. Write a program to sample $k$ values from the probability mass function $p_i = i/55$, $i = 1, 2, \ldots, 10$. Plot the histogram of generated values for $k = 50, 500$ and 5000. (One plot! Not three plots!)
7. Let a discrete random variable $X$ has a probability mass function $p_j = P(X = j)$. Define a new function

   $$\lambda_n = P(X = n \mid X > n - 1) = \frac{p_n}{1 - \sum_{j=1}^{n-1} p_j}.$$

   The quantities $\lambda_n$, $n \geq 1$ are called discrete hazard rates since if we think of $X$ as the lifetime of some item then $\lambda_n$ represents the probability that an item has reached the age $n$ will die before $n + 1$. If we are given the mass function $p$, we can simulate $X$. Write a program to simulate $X$ when only $\lambda_n$s are given. For the mass function $p_j = (0.9)^j$, first compute the hazard function $\lambda_n$ for $n = 1, 2, \ldots, 100$. Then, use these $\lambda_n$'s in your program to simulate 500 values of $X$. Plot a histogram of these simulated values.
8. Derive and implement a method to generate samples of a Weibull random variable whose probability distribution function is given by

   $$F(x) = 1 - \exp(-\alpha x^{\beta}), \quad 0 < x < \infty.$$

   Run your program to simulate 1000 values of Weibull random variable with $\alpha = 1$ and $\beta = 0.5$.

9. Write a program that uses the rejection method to sample a random variable having the distribution function:

$$F(x) = \int_0^\infty x^y \exp(-y) dy, \quad 0 \le x \le 1.$$

10. Suppose it is easy to generate a random variable from any of the distributions $F_i$, $i = 1, 2, \ldots, k$. How can we generate a random variable from the distribution:

$$F(x) = \prod_{i=1}^{k} F_i(x).$$

Hint: If $X_i$s are random variables with distributions $F_i$s, respectively, then what random variable $X$ has the distribution $F$?