

Theorem

Let X and X' be $(K\text{-equiv})$ smooth v.

and assume that the pluricanonical system is free.

$$\chi(X, K_X^{\otimes r} \otimes \Omega_X^k) = \chi(X', K_{X'}^{\otimes r} \otimes \Omega_{X'}^k)$$

$\forall r, k \in \mathbb{Z}$. *Remark: In all places can replace X by any h^P the only problem is that now for ∞ set of r does not immediately implies validity for all $r \in \mathbb{Q}$!*

pf: the statement is true for curves and surfaces. since $K\text{-equiv} \Rightarrow$ isom. in codim 1.

By induction, suppose true for all dim $\leq h-1$. let $\dim X = \dim X' = h$.

In fact can take $D \mapsto D'$ for any pair of $K\text{-equiv}$ div. i.e. By Serre duality, may assume that $k \geq 0$. since they are poly in r , so if prove for an set of number r , then done. (for any $r \in \mathbb{Q}$!)

Suppose $sK_X, sK_{X'}$ are free. let $D \in |sK_X|, D' \in |sK_{X'}|$ be corr. divisor. (smooth)

(then so is $K_X|_D \rightarrow K_{X'}|_D$ as line bundle so usually may think first prove in div case then ext. to line bundle).

$$K_D = (K_X + D)|_D = (s+1)K_X|_D$$

$$K_{D'} = (K_{X'} + D')|_{D'} = (s+1)K_{X'}|_{D'}$$

$$\varphi^* K_D = (s+1) \varphi^* K_X|_D \cong (s+1) \varphi'^* K_{X'}|_{D'} = \varphi'^* K_{D'}$$

so D, D' are still $K\text{-equiv}$.

notice. \nexists ample $K\text{-equiv}$ divisors. claim: $\chi(X, \Omega_X^k(-lD)) = \chi(X', \Omega_{X'}^k(-lD')) \quad \forall l \geq 0$

pf: for $l=0$, this follows from the equivalence of Hodge numbers.

(proved via motivic integration)

for $l=1$. consider

$$0 \rightarrow \Omega_X^k(-D) \rightarrow \Omega_X^k \rightarrow \Omega_X^k|_D \rightarrow 0$$

ie. only need to show the RHS 2 terms are the same for D & D' .

Again this is by the result for $\dim = n-1$

bec. any $\Omega_D^k \otimes \mathcal{O}_D(\lambda D) = \Omega_D^k \otimes K_D \left(-\frac{\lambda S}{S+1} \right)$

It is OK. \square . $\in \mathbb{Q}$

this show that all smooth K -equiv. v. with free pluri-can. system have the same "twisted χ_y " genus!

ie. all Chern numbers $c_i \cdot \chi_{\mathbb{R}}^{n-i}$ are the same.

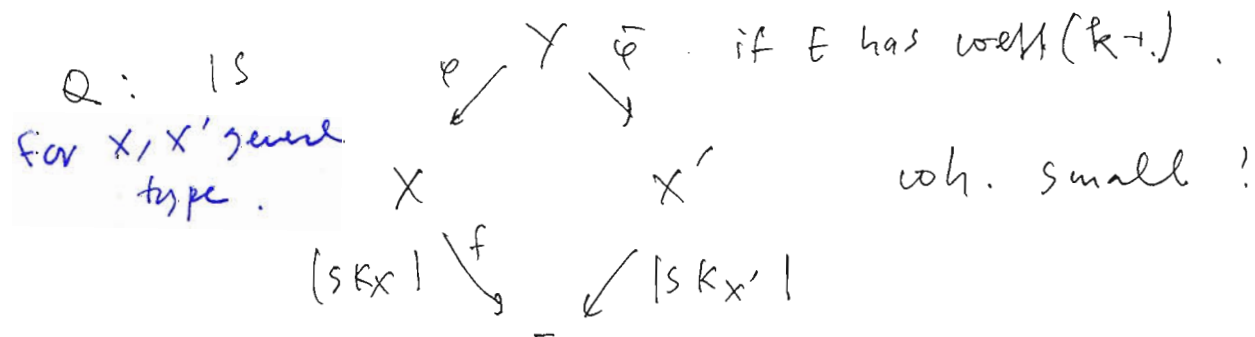
$$\chi_{\mathbb{R}}^k := \text{ch}(\Omega^k), \text{td}(X)$$

this generalises totaro's result, in the case of good smooth min. models.

Notice that k negative means $\Lambda^k T$.

in particular, $\chi(X, T)$ allowed.

- How to remove the minimal model assumption?
- Can one prove $\chi(X, \Omega_X^k) = \chi(X', \Omega_{X'}^k)$ directly without using equiv. of Hodge numbers?
- Instead of $\Lambda^k(\Omega)$, how about $S^k(\Omega)$?



then $k = \text{codim of } \varphi(E) \text{ in } X$. so fiber $\dim = n-1-k$.
 (Wisniemski $\Rightarrow n-1 + \dots + 0!$) if $\dim \text{ fiber } f = 2a$ (real dim)
 \rightarrow only for extr. rays? $? 2n - 2(n-k-a) > 4a$
 ie. $k+a > 2a$ ie $k > a$?