

Department of Mathematics  
National Taiwan University  
Taipei, Taiwan 10617

Email: [chern@math.ntu.edu.tw](mailto:chern@math.ntu.edu.tw)  
Website: <http://www.math.ntu.edu.tw/~chern>

### **Academic Interests**

- Partial Differential Equations, Continuum Mechanics, Scientific Computing, Fast Algorithms, Image Processing, Compressive Sensing, Numerical Optimization

### **Education**

- Ph.D., Mathematics, New York University, 1983.
- M.S., Mathematics, National Taiwan University, 1978.
- B.S., Mathematics, National Taiwan University, 1975.

### **Experience**

- Scientific Advisor, Moldex3D Company, Zhubei City, Taiwan (2018-2022)
- Professor Emeritus, Dept. of Math., National Taiwan University (2018-)
- Chair Professor, Dept. of Math., National Central Univ. (2014-2015)
- Director, Center of Mathematical Modeling and Scientific Computing, National Chiao Tung University (2012-2014)
- Chairman, Dept. of Math., National Taiwan University (1999-2002)
- Professor, Dept. of Math., National Taiwan University (1991-2014, 2015-2018)
- Assistant Mathematician, Math & Comput. Sciences, Argonne National Lab. (1989-1991)
- Research Assistant Professor, Courant Institute, New York Univ. (1987-1989)
- Associate Research Fellow, Inst. of Math., Academia Sinica (1983-1989).

### **Academic Visits**

- Mechanical Engineering, UC Berkeley & Lawrence Berkeley National Laboratory, Spring 1984.
- Mathematics, University of Pittsburgh, Spring 1986.
- Mathematics, Chinese University of Hong Kong, Spring 2008, Fall 2016.
- Oden Institute for Computational Engineering and Sciences, University of Texas at Austin, summer 2008.

### **Awards**

- Fellow, Taiwan Society for Industrial and Applied Mathematics, 2020.
- Society Award, The Mathematical Society of the Republic of China, 2018.
- Chair Professor, National Central Univ. 2014-2015.
- Distinguished Professor, National Chiao Tung University, 2012-2014.
- Distinguished Professor, National Taiwan University, 2008-2011.
- Outstanding Research Award, National Science Council, Republic of China, 1992-1993.

## Professional Services

- President of the Taiwan Society for Industrial and Applied Mathematics, 2012-2015, 2015-2018
- Vice President of the Chinese Mathematical Society, 2002-2004.
- Executive committee member, Scientific committee member, Asia section, Society of Industrial and Applied Mathematics, 2005-2014.
- Advisory committee member, Scientific committee member, Asia section, Society of Industrial and Applied Mathematics, 2014-now.

## Editorial board

- Journal of Computational Mathematics (**SCI**), associate editor, 2006-2016
- East Asian Journal on Applied Mathematics (**SCI**), associate editor, 2011-2016.
- East Asian Journal on Applied Mathematics (**SCI**), honorary editor, 2016-now.
- International Journal of Numerical Analysis & Modeling, Series B (**SCI**), associate editor, 2011-2013
- Communications in Information and Systems (ESCI), editor, 2014-now.

## Ph.D Students

- Yuguo Liu, A majorization-minimization algorithm for electrical impedance tomography, 2018.
- Yi-Cheng Hsu, Mathematical theory and technical designs for magnetization excitation in high field MRI, 2013.
- Li-Ren Lin, Mass Redistribution and Its Applications to the Ground States of Spin-1 Bose-Einstein Condensates, 2013.
- Yu-Chen Shu, A Coupling Interface Method for Elliptic Interface Problems, 2007.
- Jengnan Tzeng, Subspace Watermarking Method for Copyright Protection: Symmetric and Asymmetric, 2003.
- Chien-Chang Yen, Difference Wavelets and Wavelets on Intervals, 1999.

## Master Students: 34

1. Yu-Ting Hong, Numerical Simulations of Elastic Flows in Eulerian Coordinate Using WENO Scheme, 2021.
2. Yu-Hsiang Chen, Isogeometric Analysis for Solving Elliptic Equations in a Geometric Lens, 2019.
3. Hsu, Ching-Kai, Toeplitz MUSIC Algorithm for Spectral Estimation Problem, 2018. (Co-advisor: Ming-Cheng Shiue, 2018).
4. Huang, Yu-Hao, Microarray image processing and segmentation, 2018.

5. Chien-Tsung Huang, Dynamic Foreground Detection and Tracking from Video using Markov Random Field, 2016.
6. Yung-Hsien Lu, Numerical Simulations of Soliton Collisions in Two-component Bose-Einstein Condensates, 2015.
7. Ssu-Han Liu, Solving Some Moving Interface Problems by the Coupling Interface Method, 2013.
8. Cheng-Li Tsou, Coupling Interface Method for Solving Polyatomic Problems, 2011.
9. Hao-Pin Wu, Reconstruct Photoelastic Tomography via Augmented Lagrangian Method, 2011.
10. Yi-Man Tseng, Noise Models and Denoising Techniques, 2009.
11. Yuan-Hung Chang, Study on Surface Plasmon on Periodic Layer Structure, 2009.
12. Yu-Guo Liu, Genetic Algorithm for total variation based denoising problem, 2008.
13. Hao-Chih Lee, Interface Evolution via a Least-Square Velocity Extension: Application to Tumor Growth, 2008.
14. Chian-Song Lin, A Coupling Interface Method For Anisotropic Elliptic Problems, 2008.
15. Xian-Wen Dong, Fast Algorithm for Solving Electrostatic Potential of Macromolecules in Solvent, 2008.
16. Yu-Chun Lin, Multigrid Method for Solving Poisson-Boltzmann Equation, 2007.
17. Yi-Cheng Hsu, Strichartz Estimate for Discrete Schrodinger Equation, 2007.
18. Wen-Chi Chen, Simulating Rotating Shallow Water Flows By BGK Scheme, 2007.
19. Yu-Chin Lin, Algebraic Multigrid method of Kaczmarz method, 2007.
20. Yin-Hung Liu, Gas-Kinetic Schemes For Conservation Laws With Non-Convex Fluxes, 2007.
21. Yu-Shiuan Tsai, Two-Level preconditioners for Image Restoration Problem, 2005.
22. Jui-Peng Chang, Numerical method of the Quantum Drift Diffusion in Semiconductor, 2005.
23. Yu-Chen Shu, Interface Problem and Algebraic Multigrid Method, 2004.
24. Pei-Ru Chen, A Fictitious Domain Method for Poisson Equation on General Domain, 2002.
25. Chao-Mei Liu, Multigrid Method for Elliptic Equations with Discontinuous Coefficients, 2001.
26. Hsiao-Wei Huang, A Numerical Method for Valuing Two-Factor Convertible Bonds, 2000.
27. Ming-Cheng Shiue, A Boundary Element for Two-Dimensional Linearized Poisson-Boltzmann Equation, 2000.
28. Chung-min Lee, Domain-Compactization Method and Its Applications, 2000.
29. Guo Ming-Huang, Decomposition of wavelet filters and its application to image compression, 1999.
30. Shih-Ting Huang, Image Reconstruction for Tomography, 1998.
31. Hung Che Chern, Vector Spectral Methods for the shallow water equations on the sphere (coadvisor Sze-Bi Hsu) 1993.

32. Bo-Jian Li, Some numerical methods for computing shock waves, 1987.
33. Guo-Ging Huang, On the viscous shock of system of viscous conservation laws, 1987.
34. Keh-Ming Shyue, On quasi one-dimensional gas dynamics, 1986.

## JOURNAL PUBLICATIONS

- [J1] Liren Lin and IL Chern, “Phase transition between two-component and three-component ground states of spin-1 Bose-Einstein condensates,” *Bulletin of the Institute of Mathematics Academia Sinica (New Series)* Vol. 20 (2025), No. 2, pp. 131-158.
- [J2] IL Chern, CF Chou and TT Shieh, “Ground-state patterns and phase diagram of spin-1 Bose-Einstein condensates in uniform magnetic field,” *Physica D: Nonlinear Phenomena* 388 (2019) 73-86. (SCI IF 1.810).
- [J3] IL Chern, M Mei, X Yang and Q Zhang, “Stability of non-monotone critical traveling waves for reaction-diffusion equation with time-delay,” *Journal of Differential Equations* **259** (Aug. 2015) 1503-1541. (SCI IF 1.680, Math 16/310 Q1).
- [J4] J-H Chen, I-L. Chern\* and W.W. Wang, “A Complete Study of the Ground State Phase Diagrams of Spin-1 Bose-Einstein Condensates in a Magnetic Field via Continuation Methods,” *Journal of Scientific Computing*, **64** (July, 2015) 35-54. (SCI IF 1.698, Applied Math 23/255 Q1).
- [J5] Y-C Shu, I-L. Chern\* and C-C Chang, “Accurate Gradient Approximation for Complex Interface Problems in 3D by an Improved Coupling Interface Method,” *Journal of Computational Physics* **275** (2014) 642-661. (SCI IF 2.138, Phys, Mathematical 3/54 Q1, Computer science, interdisciplinary applications 18/102 Q1).
- [J6] Liren Lin and I-Liang Chern\*, “A kinetic energy reduction technique and characterizations of the ground states of spin-1 Bose-Einstein condensates,” *Discrete and Continuous Dynamical Systems, Ser. B*, **19**(4) (2014) 1119-1128. (SCI IF 1.005 Mathematics 83/310 Q2, Math, Applied 129/255 Q3).
- [J7] E. S. Helou, Y. Censor\*, T-B Chen, I-L Chern, ÁR De Pierro, M Jiang and H H-S Lu, “String-averaging expectation maximization for maximum likelihood estimation in emission tomography,” *Inverse Problems* **30**, 5 (2014) 055003 (SCI IF 1.323, Mathematics, Applied 54/255 Q1).
- [J8] B.-W. Jeng, C.-S. Chien\* and I-L. Chern, “Spectral collocation and a two-level continuation scheme for dipolar Bose-Einstein condensates,” *Journal of Computational Physics* **256** (2014) 713-727. (SCI IF 2.138, Phys, Mathematical 3/54 Q1, Computer science, interdisciplinary applications 18/102 Q1)
- [J9] Chern, I-Liang\* and Hai-Liang Li, “Long-time behavior of the nonlinear Schrödinger–Langevin equations,” *Bulletin of the Institute of Mathematics, Academia Sinica*, Vol. 8, 505-544 (2013).
- [J10] Yi-Cheng Hsu, I-Liang Chern, Wei Zhao, Borjan Gagoski, Thomas Witzel, and Fa-Hsuan Lin\*, “Mitigate B11 Inhomogeneity Using Spatially Selective Radio-frequency Excitation with Generalized Spatial Encoding Magnetic Fields,” *Magnetic Resonance in Medicine* **71**: 1458-1469 (2014). (SCI IF 3.571, Radiology, nuclear medicine & Medical imaging 20/125 Q1)

- [J11] Weizhu Bao, I-Liang Chern and Yanzhi Zhang\*, "Efficient methods for computing ground states of spin-1 Bose-Einstein condensates based on their characterizations," *Journal of Computational Physics* **253** (2013) 189-208. (SCI IF 2.310, Phys, Mathematical 3/54 Q1, Computer science, interdisciplinary applications 18/102 Q1)
- [J12] Y. Li, I-L. Chern, J-D. Kim, X. Lin\*, "Numerical method of fabric dynamics using front tracking and spring model," *Comm. in Comput. Physics*, Vol. 14, No. 5 (Nov. 2013) 1228-1251. (SCI IF 2.077, Phys, Mathematical 10/54 Q1)
- [J13] P. Chen\*, C. Lin and I. Chern, "A Perfect Match Condition for Point-Set Matching Problems Using the Optimal Mass Transport Approach," *SIAM J. on Imaging Sciences*, 6 (2) (2013) 730-764 (SCI IF 4.656, Math, applied 11/255 Q1).
- [J14] Chun-Hao Teng, I-Liang Chern and Ming-Chih Lai\*, "Simulating binary fluid-surfactant dynamics by a phase field model," *Discrete and Continuous Dynamical Systems - Series B*, Vol 17, No 4, 1289-1307 (2012). (SCI IF 0.921, Math, applied 148/255 Q3)
- [J15] Yongguei Zhu\* and I-Liang Chern, "Convergence of the alternating minimization method for sparse MR image reconstruction," *Journal of Information & Computational Science* 8:11 (2011) 2067-2075.
- [J16] Jen-Hao Chen, I-Liang Chern\*, Weichung Wang, "Exploring Ground States and Excited States of Spin-1 Bose-Einstein Condensates by Continuation Methods," *Journal of Computational Physics*, Vol. 230, (2011), 2222-2236.(SCI IF 2.310, Phys, Mathematical 3/54 Q1, Computer science, interdisciplinary applications 18/102 Q1)
- [J17] Daomin Cao, I-Liang Chern, Jun-Cheng Wei\*, "On Ground State of Spinor Bose-Einstein Condensates," *NOEDA-Nonlinear Partial Differential Equations and Applications* Vol. 18, No. 1, (2011), 427-445.(SCI IF 0.897, Math, applied 109/255 Q2)
- [J18] I-Liang Chern and Chun-Hsiung Hsia\*, "Dynamic phase transition for Cahn-Hilliard equations in cylindrical geometry," *Discrete and Continuous Dynamical System, B*, Vol. 16, No. 1 (2011), 173-188.(SCI IF 0.921, Math, applied 148/255 Q3)
- [J19] Yu-Chen Shu, Chiu-Yen Kao, I-Liang Chern\*, Chien C. Chang, "Augmented Coupling Interface Method for Solving Eigenvalue Problems with Sign-changed Coefficients," *Journal of Computational Physics*, Vol. 229 (2010), 9246-9268.(SCI IF 2.310, Phys, Mathematical 3/54 Q1, Computer science, interdisciplinary applications 18/102 Q1)
- [J20] Chang, Chien-Cheng\*, Yu-Chen Shu and I-Liang Chern, "Solving guided wave modes in plasmonic crystals," *Phys. Rev. B* **78**, 035133 (2008). (SCI IF 3.736, Phys, condensed matter, 14/64 Q1)
- [J21] Chern, I-Liang\* and Yu-Chen Shu, "Coupling interface method for elliptic interface problems," *Journal of Computational Physics*, Vol. 225, No. 2, pp.2138-2174 (2007). (SCI IF 2.310)
- [J22] Bao, Weizhu\*, I-Liang Chern and Fong Yin Lim, "Efficient and spectrally accurate numerical methods for computing ground and first excited states in Bose-Einstein condensates," *Journal of Computational Physics*, no. 2, pp. 836-854 (2006).(SCI IF 2.310)
- [J23] Tzeng, Jengnan, Wen-Liang Huang\* and I-Liang Chern, "An asymmetric subspace watermarking method for copyright protection," *IEEE Transactions on Signal Processing*, Vol. 53, No. 2, pp. 1-9(2005). (SCI IF 2.628)
- [J24] I-Liang Chern, Jian-Guo Liu\* and Wei-Cheng Wang, "Accurate Evaluation of Electrostatics for Macromolecules in Solution," *Methods and Applications of Analysis*, Vol. 10, No. 2, pp. 309-328 (2003)

- [J25] Chang, Qiangshun and I-Liang Chern\*, “Acceleration methods for total variation-based image denoising,” *SIAM J. Sci. Comp.*, Vol. 25, No. 3, pp. 982-994 (2003). (SCI IF 1.569)
- [J26] Zhilin Li, Wei-Cheng Wang, I-Liang Chern and Ming-Chih Lai, “New formulation for interface problems in polar coordinates,” *SIAM J. Sci. Comp.*, Vol. 25, No. 1, pp. 224-245 (2003). (SCI IF 1.569)
- [J27] Tzeng, Jengnan, Wen-Liang Huang\* and I-Liang Chern, “Enhancing image watermarking methods with/without reference images by optimization second-order statistics,” *IEEE Transactions on Image Processing*, Vol. 11, No. 7, pp. 771-782(2002). (SCI IF 3.042)
- [J28] Chern, I-L.\* and C.-C. Yen, “Difference wavelet – theory and a comparison study,” *Methods and Applications of Analysis*, Vol. 9, No. 4, pp. 469-492 (2002).
- [J29] Chern, I-L.\*, “Local and global interaction for nongenuinely nonlinear hyperbolic systems of conservation laws,” *Indiana University Mathematics Journal*, **49** No. 3 (2000) 1199-1228. (SCI IF 1.10)
- [J30] Chern, I-L.\* and Ming Mei, “Asymptotic stability of critical viscous shock wave for a degenerate hyperbolic conservation law,” *Comm. Partial Differential Equations* **23** (1998) 869-886 (SCI IF 0.894)
- [J31] Chern, I-L.\*, “Long-time effect of relaxation for hyperbolic conservation laws,” *Comm. Math. Phys.* **172** (1995) 39-55. (SCI IF 1.941)
- [J32] Yang, X.-L., I-L. Chern, N. Zabusky, R. Samtaney and J. Hawley, “Vorticity generation and evolution in shock-accelerated density-stratified interfaces,” *Phys. Fluid A* **4**, no. 7 (1992) 1531-1540. (SCI IF 1.926)
- [J33] Chern, I-L. and I. Foster, “Parallel implementation of a control method for solving PDEs on the sphere,” *Parallel Processing for Scientific Computing*, 301-306, SIAM, Philadelphia, PA, (1992).
- [J34] Chern, I-L., T. Colin and H. Kaper, “Classical solutions of the nondivergent barotropic equations on the sphere,” *Comm. in Partial Differential Equations* **17** no.5 & 6, (1992) 1001-1019. (SCI IF 0.894)
- [J35] Chern, I-L., “Multiple-mode diffusion waves for viscous nonstrictly hyperbolic conservation laws,” *Comm. Math. Phys.* **138** (1991) 51-61. (SCI IF 1.941)
- [J36] Chern, I-L., “Large-time behavior of solutions of Lax-Friedrichs finite difference equations for hyperbolic systems of conservation laws,” *Math. Comp.* **56**, no. 193 (1991) 107-118. (SCI IF 1.313)
- [J37] Yang, X.-L., N. Zabusky, and I-L. Chern, “Breakthrough via Vortex dipoles in shock accelerated density stratified layers,” *Phys. Fluid A* **2** no.6 (1990), 892-895. (SCI, 1.926)
- [J38] Chern, I-L., “Stability theorem and truncation error analysis for the Glimm scheme and for a front tracking method for flows with strong discontinuities,” *Comm. Pure Appl. Math.* **42** (1989), 815-844. (SCI IF 2.575)
- [J39] Chern, I-L. and T.-P. Liu, “Erratum, Convergence to diffusion waves of solutions for viscous conservation laws,” *Comm. Math. Phys.* **120** (1989), 525-527. (SCI IF 1.941)
- [J40] Chern, I-L. and T.-P. Liu, “Convergence to diffusion waves of solutions for viscous conservation laws,” *Comm. Math. Phys.* **110** (1987), 503-517. (SCI IF 1.941)

- [J41] Chern, I-L. and P. Colella, A conservative front tracking method for hyperbolic conservation laws, UCRL-97200, Lawrence Livermore National Laboratory, Livermore, CA, (1986)
- [J42] Chern, I-L., J. Glimm, O. McBryan, B. Plohr, and S. Yaniv, "Front tracking for gas dynamics," *J. Comp. Phys.* **62** (1986) 83-110. (SCI IF 2.310)
- [J43] W. Miranker and I-L. Chern, "Dichotomy and conjugate gradients in the stiff initial value problem," *Lin. Alg. Appl.* **36** (1981)(SCI IF 0.974)

## BOOKS

- [B1] YL Zhu, XN Wu, IL Chern and ZZ Sun, *Derivative Securities and Difference Methods*, 2nd edition, Springer Finance (2013) pp. 1-647.
- [B2] You-lan Zhu, Xiaonan Wu and I-Liang Chern, "Derivative Securities and Difference Methods," Springer, (2004) pp. 1-513.

## LECTURE NOTES

- [L1] I-Liang Chern, *Financial Mathematics*, pp.1-108, (1998, 2000).
- [L2] I-Liang Chern, *Mathematical Modeling and Ordinary Differential Equations*, pp. 1-225, (2002, 2015).
- [L3] I-Liang Chern, *Finite Difference Methods for Partial Differential Equations*, pp. 1-121 (2004).
- [L4] I-Liang Chern and Jun Zou, *Computational and Applied Mathematics*, pp. 1-158 (2010).
- [L5] I-Liang Chern, *Methods in Applied Mathematics* pp. 1-71 (2012).
- [L6] I-Liang Chern, *Applied Analysis*, pp. 1-176 (2013).
- [L7] I-Liang Chern, *Computational Mathematics*, pp. 1-160 (2015)
- [L8] I-Liang Chern, *A Supplementary Note on Discrete Differential Geometry*, pp. 1-166 (2019)
- [L9] I-Liang Chern, *Convex Optimization*, pp. 1-120 (2018).
- [L10] I-Liang Chern, *Linear Algebra and Its Applications*, pp. 1-200 (2021).
- [L11] I-Liang Chern, *Numerical Partial Differential Equations*, pp. 1-235 (2023).
- [L12] I-Liang Chern, *Fundamentals of Continuum Mechanics*, pp.1-367 (2020,2024).

## Selected Invited Talks: 2009-2016

1. Mathematical Theory for Ground States of Spin-1 Bose-Einstein Condensates, Institut für Mathematik, TU Berlin, Germany, October 27, 2017.
2. Mathematical Theory for Ground States of Spin-1 Bose-Einstein Condensates, The 25th Annual Workshop on Differential Equations, National Chiao Tung University, Jan. 7-8, 2017.
3. Mathematical Theory for Ground States of Spin-1 Bose-Einstein Condensates, Southern University of Science and Technology of China, December 7, 2016.

4. Mathematical Theory for Ground States of Spin-1 Bose-Einstein Condensates, Hong Kong Baptist University, Hong Kong, Nov. 1, 2016.
5. Ground State Patterns and Phase Transitions for Spin-1 Bose-Einstein Condensates, Numerics and Analysis, Hong Kong University of Science and Technology, Hong Kong, Oct. 25, 2016.
6. Mathematical Theory for Ground States of Spin-1 Bose-Einstein Condensates, Plenary speaker, International Congress for Chinese Mathematicians, Beijing, August 10, 2016.
7. Mathematical Theory for Ground States of Spin-1 Bose-Einstein Condensates, Capital Normal University, Beijing, August 8, 2016.
8. Ground State Patterns and Phase Transitions for Spin-1 Bose-Einstein Condensates, East Asia Section of SIAM 2016, Macau, June 22, 2016.
9. Ground State Patterns and Phase Transitions for Spin-1 Bose-Einstein Condensates, Numerics and Analysis, Caltech, February 3, 2016.
10. Ground State Patterns and Phase Transitions for Spin-1 Bose-Einstein Condensates, Numerics and Analysis, Claremont-McKenna College, January 29, 2016.
11. Ground State Patterns and Phase Transitions for Spin-1 Bose-Einstein Condensates, Numerics and Analysis, International Workshop on Computational Mathematics, June 29-July 2, 2015, Qingdao, China.
12. Ground State Patterns and Phase Transitions for Spin-1 Bose-Einstein Condensates, Numerics and Analysis, Workshop on High Performance and Multiphase/Complex Fluids (2 - 6 Mar 2015), Institute of Mathematical Sciences, National University of Singapore.
13. Ground State Patterns and Phase Transitions for Spin-1 Bose-Einstein Condensates, The 5th International Conference on Scientific Computing and Partial Differential Equations (SCPDE14) - On the Occasion of Eitan Tadmor's 60th Birthday, Dec. 8-12, 2014, Hong Kong Baptist University, HK.
14. Ground State Patterns and Phase Transitions for Spin-1 Bose-Einstein Condensates, The third cross-strait conference on computational mathematics, Sep. 19-22, 2014, Xiangtan University, China.
15. Ground State Patterns and Phase Transitions for Spin-1 Bose-Einstein Condensates, Annual Meeting of Japan Society for Industrial and Applied Mathematics, 3 September 2014, Tokyo, Japan.
16. Ground States of Spin-1 Bose-Einstein Condensates w/o external magnetic field, International Conference on the Mathematical Theory of Liquid Crystals and Related Topics, 19 June 2014, NYU-Shanghai, China.
17. Ground States of Spin-1 Bose-Einstein Condensates w/o external magnetic field, 2014 Japan-Taiwan Joint Workshop on Numerical Analysis and Scientific Computation 6 April 2014, Kyoto University, Japan.
18. On Ground States of Spin-1 Bose-Einstein Condensates w/o external magnetic field, IMS Workshop on Nonlinear PDEs from Fluids and Related Topics, 24-26 March 2014, Chinese University of Hong Kong, Hong Kong.
19. On Ground States of Spin-1 Bose-Einstein Condensates w/o external magnetic field, 2013 NCTS Mathematics Physics Joint Colloquium, Sep. 26, 2013, Taipei.

20. On Ground States of Spin-1 Bose-Einstein Condensates with/without external magnetic field, International Congress of Chinese Mathematicians, July 14-19, 2013, Taipei.
21. Exploring Ground States and Excited States of Spin-1 Bose-Einstein Condensates, Seminar talk at Claremont McKenna College, USA, Jan. 23, 2013.
22. Exploring Ground States and Excited States of Spin-1 Bose-Einstein Condensates, Seminar talk at California State University at Long Beach, USA, Jan. 24, 2013.
23. Exploring Ground States and Excited States of Spin-1 Bose-Einstein Condensates with/without External Magnetic Field, Workshop on “Confined Quantum Systems: Modeling, Analysis and Computation”, Feb. 4 - 8, 2013, Wolfgang Pauli Institut, Vienna, Austria.
24. Coupling Interface Method and Macromolecules in Ionic Solution, A workshop in honor of Stanley Osher, December 15-18, 2012, Tsing Hua, Beijing, China.
25. Coupling Interface Method and Macromolecules in Ionic Solution, Cross Strait Conference on Applied Mathematics, April 7-11, 2012, Ning Bo, China.
26. Exploring ground states and excited states of Spin-1 Bose-Einstein condensates, International Conference on Mathematical Modeling, Analysis and Computation, Xiamen, June 22-25, 2012.
27. Accurate Gradient Approximation at Interfaces by Coupling Interface Method for Elliptic Interface Problems, 2011 International Conference on Applied Mathematics and Interdisciplinary Research, Chern Institute of Mathematics, Nankai Univ., June 13-16, 2011.
28. Coupling interface method, methods and applications, The 2011 Workshop on Scientific Computing, Dec. 10-12, Macau, 2011
29. Characterization of ground states of spin-1 Bose-Einstein condensates, The seventh International Congress on Industrial and Applied Mathematics, Vancouver, July 18-22, 2011.
30. Exploring ground states and excited states of Spin-1 Bose-Einstein condensates by Continuation method, International Conference on Applied Mathematics, Hong Kong, June 7-11, 2010
31. Parallel MR imaging, SIAM Conference on Image Science, Chicago, Apr. 12-14, 2010
32. Two Finite Difference Methods for Solving Poisson-Boltzmann Equation, International Workshop on Continuum Modeling of Biomolecules, Beijing, September 14-16, 2009.

## Research Summary

**Bose-Einstein Condensates and Nonlinear Schrödinger equations** In the past ten years, my major research topic is on exploring the ground states and vortex states of spinor Bose-Einstein condensations (BEC), both numerically and analytically. The spinor BECs involve coupled nonlinear Schrödinger systems.

Joint with J-H Chen and W Wang [16], we developed a pseudo-arc length continuation method to compute ground states and excited states for spinor BECs. We characterized ground-state patterns and observed phase separation for excited states in ferro-magnetic systems [16]. Later, we provided a complete numerical study of the ground-state patterns and phase transitions for spin-1 BECs in uniform magnetic field [4].

Inspired by our computational simulations, I began analytic studies of the ground-state patterns and their phase transitions. With Cao and Wei [17], we obtained existence and non-existence theorems for ground states of spin-1 BECs in one space dimension. Later, with my Ph.D student,

Liren Lin [6], we showed that the ground-state pattern in ferromagnetic systems has to be constant multiples of a scalar field, called single mode approximation (SMA) in physics literature, whereas it becomes a two-component BEC for antiferromagnetic systems. The key step is a kinetic-energy reduced mass-redistribution lemma, which is new and important. We used it to develop a fast algorithm for computing ground states of spin-1 BECs in Ioffe-Pritchard magnetic field and discovered that its ground state has to be SMA as long as the I-P field exists [11].

Later, with Liren Lin, we applied this lemma and discovered more analytic properties of ground states for antiferromagnetic spinor BECs in uniform magnetic field in three dimensions. As a consequence, we proved a global bifurcation theorem for ground state from 2C state to 3C state as the applied magnetic field increases, a phenomenon that was observed both experimentally and numerically [R1]. A more recent one is a complete analytic theory for ground states in Thomas-Fermi regime and semi-classical regime, a joint work with T-T Shieh and CF Chou [R2].

There are three side works related to my research on PDEs. One is a joint work with B-W Jeng and C-S Chien [8], where we proposed a spectral collation method and a two-level continuation method to study the ground states and vortex states of dipolar BECs. Extensive numerical experiments in 3D were reported.

The other side work was jointly with Hailiang Li [9] on the asymptotic behaviors of the solutions of the Schrödinger-Langevin equation. It is shown that the momentum damping overwhelms the quantum dispersion and the solution tends pointwisely to a nonlinear diffusion wave. The new analytic trick, which is different from my earlier work on diffusion wave, is an energy estimate associated with the quantum dispersion.

A recent one is a stability analysis for non-monotone critical traveling waves for a time-delayed reaction-diffusion equation arisen from biology. It exhibits oscillatory traveling wave if the time-delay is long or the traveling speed is fast. The stability of such oscillatory traveling waves, especially the critical case, is challenging. By using a weighted energy method, we were able to give a positive answer. This is a joint work with my previous post-doctor Ming Mei [3].

**Interface Problems and Applications** This research direction was motivated from the study of protein-peptide interaction for drug design. Joint with Liu and Wang, we proposed a fast algorithm and a desingularization technique for solving Poisson-Boltzmann equation with discontinuous coefficients[24]. Later, with Y-C Shu, we proposed the coupling interface method (CIM) to solve the elliptic interface problems, where the elliptic coefficients are discontinuous [21]. Our method is for arbitrary dimensions. It is second order accurate not only for the unknown  $u$ , but also for its gradient. It is also capable to handle high contrast and complex interface problems which are known to be very difficult numerically. Our numerical results show that this method is superior to many other computational interface methods. This work has attracted much attention in this field.

A continuation of this work was a joint work with the front tracking team from Stony Brook, where we had merged our 3D CIM code with their FronTier Code to study fabric dynamics [12]. Another work was an improvement of CIM. In applying our original CIM for very complex interfaces in 3D, the absolute errors fluctuate at different mesh sizes, although the overall error is still second order through least squares fitting. In this improved work [5], two recipes were proposed to cure this problem, thereby CIM can handle quite complex interfaces in three dimensions without error fluctuation. We got an impressive comment from referees.

We have applied CIM to surface plasma problems [20, 19] and ecological habitat pattern problems [19]. Surface plasmonic waves are EM waves propagating near the metal-dielectric interfaces. They are important for biosensor design, superlens study, etc. in optical science and engineering. It is difficult to compute these waves due to their confined and oscillatory behaviors near the interface.

Using CIM, we were able to compute them and analyze their optical properties. The ecological habitat pattern problem is an elliptic eigen-value problem with sign-changed coefficient, where CIM was suitable applied and gave impressed results.

The elliptic interface problem is a subtopic of multi-phase physics. Jointly with CH Hsia [18], we studied dynamic phase transition of binary system in cylindrical geometry. It is found that the phase separation in pan-cake geometry forms a pattern with four components, instead of two. Jointly with CH Teng and MC Lai [14], we propose a phase field model for binary fluids with surfactant. We characterize analytically the structure of interfaces, comparing with physical experiment and performing numerical simulations by spectral method. It is shown that the surfactant favors the creation of interfaces and stabilizes the formation of phase regions.

**Wavelets, Image processing and numerical optimization** I proposed “interpolating wavelets and difference wavelet”[28], which was based on interpolation/finite difference, not limited to uniform grid nor one dimension. A thorough mathematical theory was also developed.

For image denoising, jointly with QS Chang, we adopted an algebraic multigrid method to TV denoising method and got linear computational complexity[25]. Joint work with Tseng and Huang, a digital water marking method was proposed where the water mark is added in an unimportant direction of the original image, determined by singular value decomposition of the image matrix. The proposed method is robust and hard to be broken[27, 23].

In a joint work with PW Chen and C-L Lin [13], we proposed a registration method for solving point set matching problems. The method is a combination of a global affine transform and a local curl-free transform. The latter is estimated by using mass transportation theory. We applied this method to match two sets of lung branch points whose displacement is caused by lung volume changes. Nearly perfect match performances verdict the effectiveness of this model.

In a joint project with electrical engineers, my student studied how to mitigate  $B_1^+$  inhomogeneity in a high-field magnetic resonance imaging (MRI). Such inhomogeneity causes spatially dependent contrast and makes clinical diagnosis difficult. The proposed method is a two-step design procedure in which (a) a combination of linear and quadratic spatial encoding magnetic field is used to remap the  $B_1^+$  map in order to reduce the inhomogeneity problem to one dimension, (b) the locations, the amplitudes and the phases of spokes are estimated in one dimension. It is shown both numerically and experimentally that this design can mitigate the  $B_1^+$  inhomogeneity at 7T efficiently [10].

In another work with Censor, TB Chen, De Pierro, Helou, Jiang and Lu [7], we study the Expectation-Maximization (EM) algorithm for Maximum Likelihood Estimation (MLE) in Positron Emission Tomography (PET) and propose a new algorithmic structure called the String-Averaging Expectation-Maximization (SA-EM). In our simulation study, high-contrast and less noisy images with clear object boundaries are reconstructed with the proposed SA-EM algorithm in less computation time. Together with the new scheme, we propose a stopping criterion for this and other fast algorithms in tomography based on the curvature of the likelihood, as well as an L-curve to analyze iterations quality. Also, we present new convergence results for this family of algorithms.

In a joint work with Yunguei Zhu [15], we applied the alternating minimization method with total variation regularization and wavelet sparsity for sparse magnetic resonance image reconstruction. Numerical experiments were performed and convergence theorem is proved. The results show that radial compressed sensing is feasible for MR imaging.

## Computational Fluid Dynamics and Numerical PDEs

- Front tracking methods: Jointly with Glimm et al. [42](cited 340), we have developed the first successful, general-purpose front tracking code for solving gas dynamic problems such as shock diffraction problem, Rayleigh-Taylor problem, Richtmyer-Meshkov problem, etc. Jointly with P. Colella [41] (cited 141 times), we proposed the first “conservative” front tracking method for gas dynamics in two dimension. This method has been used by many researchers (e.g., Colella, Majda, Berger, etc.) to various applications (e.g., combustion flow calculations).
- Geofluid dynamics on the sphere. I proposed two versions of control-volume method on icosahedral grid for the shallow-water equations on the sphere. Its parallel implementation was also done together with I. Foster[33]. The purpose is to develop advanced fast and high resolution general circulation model for climate study. This method can achieve almost linear speedup.
- Non-divergent barotropic equations on the sphere  
The non-divergent barotropic equations on a sphere provide the simplest, yet the most important mathematical model for the description of large-scale horizontal motions of the atmosphere. Based on Hölder estimation, we proved global existence, uniqueness, and regularity theorems for the non-divergent barotropic equations[34].
- Mixing of Fluid Flows: This was a joint work with N. Zabusky. We studied the mixing process in a density-stratified fluid induced by a strike of a shock. High order Godunov method was used. A convective “breakthrough” phenomenon was observed and quantified. The phenomenon was interpreted as a dipolar-vortex dynamics. This work also gave an insight of the well-known Richtmyer-Meshkov instability in the mixing fluid flow, namely, the evolution of a shock-accelerated density-stratified interface was determined by (1) the vorticity deposition by the shock-interface interaction (zeroth order effect) and (2) the interaction of the interface with the vortex sheets shed from the transmitted and reflected waves (first order effect) [37, 32].
- I gave stability and truncation error analysis for front tracking method for hyperbolic conservation laws with strong discontinuities in one space dimension [38]. This result is the only analytical result about the front tracking method. It gives insight of wave-interaction picture near fronts.
- I identified a major error of the Lax-Friedrichs scheme was the discrete diffusion waves. They were also the time-asymptotics of the Lax-Friedrichs finite difference equations. Construction and properties of these discrete diffusion were also given [36].

### Hyperbolic Conservation Laws:

- Global existence for large initial data: I proved a global existence theorem for hyperbolic conservation laws with initial data being a perturbation of a strong discontinuity [38]. This was a generalization of Glimm’s famous work in 1965, where the flow is a perturbation of a constant state. This work answered part of the well-known problem involving the global existence of a solution for hyperbolic conservation laws with large data.  
In another work, I proved the global existence for non-genuinely nonlinear hyperbolic systems [29], where the key step is a new way to define strength of contact shocks.
- Viscous conservation laws  
In a joint work with TP Liu [40], we identified that the time-asymptotic solutions of viscous conservation laws were the diffusion waves. These waves are important because they carry the invariant masses. Optimal convergent rate to these waves was also obtained. A generalization to the nonstrictly hyperbolic systems in this direction was done by myself [35]. In this case, “multiple-mode diffusion waves” were the time-asymptotic solutions.

- Hyperbolic conservation laws with relaxation

The hyperbolic conservation laws with relaxation appear in many physical systems such as non-equilibrium gas dynamics, flood flow with friction, visco-elasticity, magnetohydrodynamics, etc. I have shown that the long-range effect of relaxation is equivalent to a viscous effect. As a consequence, the long-time behavior of solutions of such systems consist of diffusion waves. The convergent rate to these diffusion waves is also obtained [31]. This work has been cited 70 times.