國立臺灣大學數學系
九十五學年度博士班資格考試試題
科目：離散數學

2007.06.01

Each problem weights 20 points.

1. Determine the number of m-subsets of the set $[n] = \{1, 2, \ldots, n\}$ for each value of m (mod 3) and each value of n (mod 6).

2. Prove that if G is the complement of a disconnected simple graph then $e(G) \leq \Delta(G)^2$, with equality only for $K_{\Delta(G),\Delta(G)}$. Also prove that if G is a connected P_4-free graph with maximum degree k then $e(G) \leq k^2$.

3. Determine the number of spanning trees of the graph $G_{2,n} = (V_{2,n}, E_{2,n})$ with vertex set $V_{2,n} = \{(i,j) : 1 \leq i \leq 2,\ 1 \leq j \leq n\}$ and edge set $E_{2,n} = \{(i,j)(i',j') : |i - i'| + |j - j'| = 1\}$.

4. Prove that a tree T has a perfect matching if and only if $o(T - v) = 1$ for any vertex $v \in V(T)$, where $o(T - v)$ is the number of components of $T - v$ with an odd number of vertices.

5. Prove that if G is $2K_2$-free then $\chi(G) \leq \binom{\omega(G)+1}{2}$. For each positive integer k, construct a graph G_k with $\omega(G_k) = k$ and $\chi(G_k) = \binom{k+1}{2}$.