國立臺灣大學數學系

九十七學年度博士班入學考試試題

科目：幾何與拓樸

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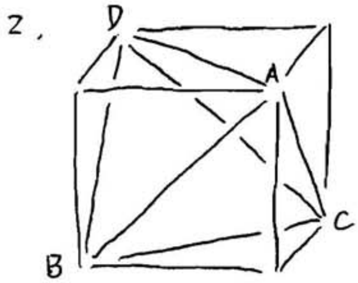
1. $C = \{ x^2+y^2-1 = 0 = z \}$ is a circle. Can you find a smooth differential 1-form $\omega = \alpha\, dx + \beta\, dy + \gamma\, dz$ in $\mathbb{R}^3 - C$ which is closed but is NOT an exact differential? If not, prove such differentials do NOT exist by deRham cohomology or any other method. If yes, $\omega = ?\,dx + ?\,dy + ?\,dz$ (25/100)

2. 

$A=(1,1,1)$, $B=(1,-1,-1)$, $C=(-1,1,-1)$, $D=(-1,-1,1)$
$K = \overline{AB} \cup \overline{AC} \cup \overline{AD} \cup \overline{BC} \cup \overline{BD} \cup \overline{CD}$, $X = \mathbb{R}^3 - K$
Find the fundamental group $\pi_1(X)$ by van Kampen's Theorem or any other method. (25/100)

3. 

Let $ds^2 = 4(dx^2+dy^2) \div (1+x^2+y^2)^2$ be the riemannian metric pulled back by the stereographic projection. $\gamma = \{ (t, 1-t) \mid 0 \le t \le 1 \}$ is a segment from $p=(0,1)$ to $q=(1,0)$. $\vec{i} = \frac{\partial}{\partial x}$ & $\vec{j} = \frac{\partial}{\partial y}$ are basis vectors. $\vec{u} = \vec{i} + \vec{j}$ is a vector at p, $\vec{v}$ is the parallel translate of $\vec{u}$ to q along γ. $\vec{v} = ?\,\vec{i} + ?\,\vec{j}$ (25/100)

4. $z = \frac{x^2}{4} + y^2$ is an elliptic paraboloid. $x - y + 2z = 4$ intersects this paraboloid in a planar curve so its torsion $\tau \equiv 0$. At the point $(x, y, z) = (2, 0, 1)$ is it correct that curvature $\kappa = \frac{37}{22}\sqrt{6}$? If not, $\kappa = ?$ (25/100)