臺灣大學數學系

九十五學年度博士班入學考試題

分析

Jun, 2006

1. Let $\{f_n\}$ be a sequence of measurable functions. Show that the set of those x such that $\{f_n(x)\}$ converges is a measurable set.

2. Show that, if $f_n \to f$ in measure and if there is an integrable function g such that $|f_n| \leq g$ for all n, then $\int |f_n - f| \to 0$.

3. Let $f(x)$ be a L^1 function on $(-\infty, \infty)$.
(a) Show that $\lim_{s\to 0} \int_{-\infty}^{\infty} |f(x+s) - f(x)|dx = 0$.
(b) Is it true that $\lim_{s\to 0} \int_{-\infty}^{\infty} |f(x+sx) - f(x)|dx = 0$?

4. Let f be a L^1 function on $(-\infty, \infty)$ and $g(x) = \int_{-\infty}^{\infty} \exp\{-(y-x)^2\} f(y)dy$.
(a) Show that $g(x)$ is differentiable.
(b) Show that $g(x) \in L^p$ for all $p \geq 1$.
(c) Are the functions $\int_{-\infty}^{\infty} \exp\{-(y-x)^2\} f(y+x^2)dy$ and $\int_{-\infty}^{\infty} \exp\{-(y-x)^2\} f(yx)dy$ differentiable in x?