臺灣大學應用數學科學研究所 106 學年度碩士班甄試試題

科目：機率統計

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1. (10%) Let $P(A_i) = 1 \ \forall\ i$. Show that $P(\bigcap_{i=1}^{n} A_i) = 1$.

2. (15%) Let X and Y be mutually independent continuous random variables with the cumulative distributions $F_X(x)$ and $F_Y(y)$, respectively. Derive the distribution of X conditioning on $\{Z = 0\}$, where $Z = I(X \leq Y)$.

3. (7%) (8%) Let $T = \sum_{i=1}^{N} X_i$ and $E[N^2] < \infty$. Conditioning on $N = n$, $X_1, \cdots, X_n$ are further assumed to be independent and identically distributed with mean μ and variance σ^2. Derive the mean and variance of T.

4. (15%) Let $X_1, \ldots, X_n \overset{i.i.d.}{\sim} N(\mu, \sigma^2)$ with unknown μ and σ^2. Derive the distribution of the random quantity $\sqrt{n}(\bar{X}_n - \mu)/S_n$, where $\bar{X}_n = \sum_{i=1}^{n} X_i/n$ and $S_n^2 = \sum_{i=1}^{n}(X_i - \bar{X}_n)^2/(n-1)$.

5. (15%) Let $X_1, \ldots, X_n$ be a random sample from a geometric distribution $P(X = x) = p(1-p)^{x-1} I_{\{1,2,\ldots\}}(x)$ and p have a uniform prior distribution on $[0, 1]$. Find the Bayes estimator of p based on the loss function $L(p, \delta(X_1, \ldots, X_n)) = |\delta(X_1, \ldots, X_n) - p|$.

6. (15%) Let $X_1, \ldots, X_{n+1}$ be a random sample from $Bernoulli(\pi)$ and $h(\pi) = P(\sum_{i=1}^{n} X_i > X_{n+1} | \pi)$. Find the maximum likelihood estimator of $h(\pi)$.

7. (15%) Let $X_1, \ldots, X_n$ be a random sample from $Beta(\theta, 1)$. Find a $(1 - \alpha)$ confidence interval by inverting the likelihood ratio test for $H_0 : \theta = \theta_0$ versus $H_A : \theta > \theta_0$.