臺灣大學應用數學科學研究所

102 學年度碩士班甄試試題

科目：機率統計

1. (10%) Consider a Poisson process on the real line and denote by $N(t_1, t_2)$ the number of events in the interval (t_1, t_2). Find the conditional distribution of $N(t_0, t_1)$ given that $N(t_0, t_2) = n$, where $t_0 < t_1 < t_2$.

2. (5%) (5%) Let $Y = \ln X \sim N(\mu, \sigma^2)$. Compute the mean and the variance of X.

3. (10%) Let $X_1, \ldots, X_n$ be a random sample from a finite population $\{x_1, \ldots, x_N\}$. Find an unbiased estimator of the population variance $\sigma^2 = \sum_{i=1}^{N}(x_i - \mu)^2/N$, where $\mu = \sum_{i=1}^{N} x_i/N$.

4. (10%) (10%) Let $X_1, \ldots, X_{2m+1}$ be a random sample from the double exponential distribution $f(x|\theta) = 0.5\exp(-|x-\theta|)I(-\infty < x < \infty)$. Derive the maximum likelihood estimator of θ and its distribution.

5. (15%) Let $\widehat{\theta}_i$ be the minimum variance unbiased estimator (MVUE) of θ_i, $i = 1, \ldots, k$, and $a_1, \ldots, a_k$ be constants. Show that $a_1\widehat{\theta}_1 + \ldots + a_k\widehat{\theta}_k$ is the MVUE of $a_1\theta_1 + \ldots + a_k\theta_k$.

6. Let $X_i \sim Binomial(n_i, p_i)$, $i = 1, \ldots, m$, be independent.

(6a) (15%) Derive the likelihood ratio test for the null hypothesis $H_0 : p_1 = \ldots = p_m$ versus the alternative hypothesis $H_A : p_i \neq p_j$ for some $i \neq j$.

(6b) (5%) What is the large sample distribution of the test statistic?

7. (15%) Let $X_1, \ldots, X_n$ be a random sample from $Poisson(\lambda)$ and λ have a $Gamma(\alpha, \beta)$ distribution, where α and β are known positive constants. Find the Bayes estimator of λ under the squared error loss function.