

臺灣大學應用數學科學研究所 110 學年度碩士班甄試試題

科目：微分方程與線性代數

2020.10.23

1. (40%)(a.) Find the general solution to $\frac{dy}{dx} = \frac{3x^2y^2 - y + 1}{x - 2x^3y}$.

(b.) Solve $ty' + 2y = e^{-t}$, $y(1) = 1$ for $t > 0$.

2. (30%) (a.) Find the general solution to the following system of differential equations.

$$\begin{cases} \frac{d}{dt}x = -2y - 3z \\ \frac{d}{dt}y = x - 3y + z \\ \frac{d}{dt}z = x - 2y + 2z. \end{cases} \quad (1)$$

(b.) Determine whether the system is stable and whether the system is asymptotically stable.

3. (30%) Let \mathbb{V} be a vector space over \mathbb{C} and $\langle \cdot, \cdot \rangle$ be an inner product on \mathbb{V} . Suppose $T : \mathbb{V} \rightarrow \mathbb{V}$ is a linear mapping such that

$$\langle Tx, y \rangle = \langle x, Ty \rangle$$

for any $x, y \in \mathbb{V}$. (a.) Show that every eigenvalue of T is a real number. (b.) Show that two eigenvectors with different eigenvalues are orthogonal.