

臺灣大學應用數學科學研究所113學年度碩士班甄試筆試試題

科目：微積分

2023. 11. 02

1. (10%) Calculate  $\lim_{t \rightarrow 0} \frac{10(t - \sin t)}{t^3}$ 
  - a. (3%) State your method and explain why it works
  - b. (7%) Show the whole calculation
2. (10%) Find  $a, b \in \mathbb{R}$  such that they achieve  $\max_{a < b} \int_a^b (x - x^2) dx$ 
  - a. (3%) State your method and explain why it works
  - b. (7%) Show the whole calculation
3. (10%) Let  $y(x) = \int_0^{\sin x} \frac{dt}{\sqrt{1-t^2}}$  for  $|x| < \frac{\pi}{2}$   
Calculate  $\frac{dy}{dx}$ 
  - a. (3%) State your method and explain why it works
  - b. (7%) Show the whole calculation
4. (10%) Calculate  $\int_{\pi/4}^{\pi/2} (1 + e^{\cot \theta}) (\csc \theta)^2 d\theta$ 
  - a. (3%) State your method and explain why it works
  - b. (7%) Show the whole calculation
5. (10%) Calculate  $\int_1^{\infty} \frac{3v-1}{4v^3-v^2} dv$ 
  - a. (3%) State your method and explain why it works
  - b. (7%) Show the whole calculation
6. (10%) Let  $k \in \mathbb{N}$  arbitrarily. Find the convergence radius of the following series  $\sum_{n=0}^{\infty} \frac{(n!)^k}{(kn)!} x^n$ 
  - a. (3%) State your method and explain why it works
  - b. (7%) Show the whole calculation

7. (10%) Calculate  $\iint_A xy \sin(x^2 - y^2) dx dy$ , where

$$A = \{(x, y) : 0 < y < 1, x > y \text{ and } x^2 - y^2 < 1\}$$

- a. (3%) State your method and explain why it works
- b. (7%) Show the whole calculation

8. (15%) Let  $\alpha = \lim_{\varepsilon \rightarrow 0^+} \varepsilon^{\frac{1}{4}} \int_0^{\varepsilon^{-\frac{1}{2}}} \exp(-\varepsilon^{\frac{1}{4}} s) ds$

Which of the following is right?

- (i)  $\alpha = 0$
- (ii)  $\alpha = 1$
- (iii)  $\alpha = \infty$
- (iv) None of above

Prove your answer.

9. (15%) Let  $f: [0, 1] \rightarrow \mathbb{R}$  be continuous and satisfy  $f''(x) \geq 0$  for  $x \in (0, 1)$ . Suppose that the function  $f$  is nonconstant and there exists  $\theta \in [0, 1]$  such that  $f(\theta) = \max_{x \in [0, 1]} f(x)$ . Can  $\theta = \frac{1}{2}$  ?

Prove or disprove your answer.