臺灣大學應用數學科學研究所 109 學年度碩士班甄試試題

科目：微積分

2019.10.18

1.(25%) Use any method you know to justify the following formula:

$$\int_0^\infty \frac{t \sin tx}{(1+t^2)(4+t^2)} dt = \frac{\pi}{6}(e^{-x} - e^{-2x}), \quad x \in (0, \infty).$$

(Hint: You can try the Fourier transform.)

2.(25%) Suppose that z is a function of x and y. Let z satisfy the following differential equation

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1.$$

Please derive a general form of $z = z(x, y)$.

3.(25%) Find the minimum distance from the origin to the curve formed by the intersection of two surfaces: $x_1x_3 + x_2x_3 = -2$ and $x_1x_2 = 1$.

4.(25%) Let $\Omega = [0,1] \times [0,1]$. Prove that there exists a constant $C > 0$ such that for any $f \in C_0^1(\Omega)$

$$C \int_\Omega |f(x)|^2 dx \le \int_\Omega |\nabla f|^2 dx. \tag{1}$$

This is the well-known Poincare's inequality. What is the largest constant C for which (1) holds?