

臺灣大學數學系
九十九學年度碩士班甄試試題
科目：線性代數

2009.10.30

1. Let V be the vector space of functions on \mathbb{R} . Show that $\{\sin^n x\}_{n=0}^{\infty}$ is a linearly independent set in V . (20 pts)
2. Let V be a vector space over an infinite field F and $u_1, \dots, u_n \in V$ are linearly independent over F . Show that, for any $v_1, \dots, v_n \in V$, $u_1 + \alpha v_1, \dots, u_n + \alpha v_n$ are linearly independent over F for all but finitely many values of $\alpha \in F$. (20 pts)
3. Let V be a vector space over a field F and T a linear transformation on V . Suppose that for each $v \in V$ there exists $\lambda_v \in F$ such that $Tv = \lambda_v v$. Show that there exists $\lambda \in F$ such that $Tv = \lambda v$ for all $v \in V$. (20 pts)
4. Let F be a field and A_1, \dots, A_{n^2} a basis of $M_n(F)$, the algebra of all $n \times n$ matrices over F . Show that for any set $c_1, \dots, c_{n^2} \in F$ there exists a unique $B \in M_n(F)$ such that $\text{tr}(A_i B) = c_i$ for all $i = 1, \dots, n^2$. (20 pts)
5. Determine what kind of quadratic surface the surface represented by the equation $x^2 + y^2 + z^2 - 2xy - 2yz - 2zx = 1$ is. (20 pts)