臺灣大學數學系

八十七學年度碩士班甄試入學考試試題

高等微積分

1.
   1. Is the function $f(x)=\sqrt{x}\sin(1/x), x>0$, uniformly continuous on the interval $(0,1]$?Prove your answer.
   2. For what value of α the function $f(x)=|x|^{\alpha}\sin(1/x)$ for $x\neq 0$ and $f(0)=0$ is differentiable at $x=0$?
2. Let $x_1=c$, and for $n\geq 2$, define the sequence x_n by

$$x_n=\frac{7}{8}x_{n-1}+\frac{1}{8}.$$

   For what real value of c does the sequence x_n converge? To what limit does it converge? What can you say about the rate of convergence?(Justify your answer!)

3.
   1. Find the rectangular parallelepiped of greatest volume inscribed in the ellipsoid: $x^2/a^2+y^2/b^2+z^2/c^2=1$
   2. Describe and prove the Lagrange's method of undetermined multiplier for the extremal problem with constraint, namely, find the extreme value of a function $f(x,y,z)$ under the constraint $g(x,y,z)=0$.
4. For what values of a,b,c the integral

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{ax^2+2bxy+cy^2}dxdy$$

   converges as an improper integral. Find the value of this integral when it exists.
5. Let u a smooth solution of $u_{xx}+u_{yy}=u$ in a smooth domain D in R^2. Show that

$$\int_D u\frac{\partial u}{\partial n}\geq 0$$

   where n is the unit outer normal to the boundary ∂D. Show that equality holds if and only if $u=0$.

6.

1. Show that the function $f(x) = \tan x - x$ has for a positive integer n exactly one root $x = x_n$ in the interval $n\pi < x < (n + \frac{1}{2})\pi$.
2. Show that

$$x_n = n\pi + \frac{\pi}{2} - \frac{1}{n\pi} + o(\frac{1}{n}).$$