臺灣大學數學系

九十學年度第二學期碩博士班資格考試試題

幾何

1. (25/100) Two spaces A and B are said to have the same homotopy type if there are continuous maps $f : A \to B$ and $g : B \to A$ such that both $f(g)$ and $g(f)$ are homotopic to the identity maps. Let $A \subset \mathbb{R}^2$, $A = \{(x+2)^2 + y^2 = 1\} \cup$ $\{(-1 \le x \le 1, y = 0)\} \cup$ $\{(x-2)^2 + y^2 = 1\}$, $B =$Lemniscate $= \{r^2 = cos 2\theta\}$. Do A and B have the same homotopy type?

2. (25/100) Z $=$ arctan $(\ y \div x\ )$ $x > 0$, $y > 0$, $0 < z < \frac{\pi}{2}$. Double integral $\int_0^\infty \int_0^\infty K(x,y)\ \sqrt{1 + (\partial z/\partial x)^2 + (\partial z/\partial y)^2}\ dxdy$ is an improper integral, where $K(x,y)$ is the Gauss curvature of the graph and $dS = \sqrt{1 + (\partial z/\partial x)^2 + (\partial z/\partial y)^2}$ $dxdy$ is the surface element of surface integral. Is $\int\int\ K\ dS$ convergent or divergent? If convergent $\int\int\ K\ dS =$?

3. (25/100) $Z = \{\frac{x^2}{4} + y^2 = 1\} \subset \mathbb{R}^3$ is a cylinder, $C = Z \cap \{x + y - z = 2\}$ is a curve on Z. Is C a geodesic on Z? At the point $(x,y,z) = (2,0,0)$, geodesic curvature $K_g =$?

4. (25/100) First fundamental form $I = Edu^2 + 2Fdudv + Gdv^2 = \frac{1}{(1-u^2-v^2)^2} du^2$ $+ 0 + \frac{1}{(1-u^2-v^2)^2} dv^2$, $(1 - u^2 - v^2 > 0)$. Can you find a surface in $\mathbb{R}^3$ $x = x(u,v)$, $y = y(u,v)$, $z = z(u,v)$ having this first fundamental form? If yes, second fundamental form II $= Ldu^2 + 2Mdudv + Ndv^2 =$? Mean curvature$= H =$? Gauss curvature $= K =$?