臺灣大學數學系

八十九學年度第一學期碩博士班資格考試試題

分析

一、

Let E be a measurable set. $\int_A f = 0$ for all measurable subsets A of E. Show that $f = 0 \quad a.e. \ (E)$.

二、

$E \subset R$ is measurable. f is integrable on E. (Lebesgue)

(a)

Then so is $|f|$ and $\left| \int_E f \right| \le \int_E |f|$?

(b)

Consider the improper Riemann integral of f (of a fixed kind) on

$[a, b] \quad (a < b \le \infty)$ write $\mathbf{Imp} f = \int_a^b f$. (Riemann)

If f is Lebesgue integrable on $[a, b]$, then what conclusions can you make about $I_m f$?

三、

Let $g(t)$ be monotonic increasing and absolutely continuous on $[\alpha, \beta]$. f bounded measurable on $[a, b]$ with $a = g(\alpha), b = g(\beta)$. Show that $f(g(t))$ is measurable on $[\alpha, \beta]$ and

$$\int_a^b f(x)\ dx = \int_\alpha^\beta f(g(t))g'(t)\ dt$$

四、

Suppose that $f_k \longrightarrow f \quad a.e.$ in R and $f_k, f \in L^p(R) \quad 1 < p < \infty$. If $\|f_k\|_p \le M < \infty$, show that $\int f_k g \longrightarrow \int f g \quad \forall g \in L^q, \frac{1}{p} + \frac{1}{q} = 1$.

五、

Assume $f \in L^1(R)$ then $\lim_{n\to\infty} \sum_{k=-n^2}^{n^2} \left| \int_{\frac{k}{n}}^{\frac{k+1}{n}} f(x)\, dx \right| =$? Give reasons.

六、

(a)

Describible Cauchy's integral formula.

(b)

Show that bounded entire function (on C) reduces to constant.

七、

Calculate $\int_0^\infty \frac{\cos mx}{1+x^2}\, dx$, where $m > 0$ is a constant.