臺灣大學數學系

八十八學年度第一學期碩博士班資格考試試題

代數

1. Show that every group of order 175 is abelian.
2. Show that the additive groups of $\mathbb{R}$ and $\mathbb{C}$ are isomorphic.
3. If R is a ring (with 1), then the ideals I and J are called coprime if $I+J=R$. If I_i (i=1,2,3,4,5) are coprime in pairs, show that $I_1 I_2$ and $I_3 I_4 I_5$ are also coprime.
4. If $f: A \to A$ is an R-module homomorphism such that $f \circ f = f$, show that A = Ker $f \oplus$ Im f .
5. Let $\sigma_1, \sigma_2, \cdots, \sigma_n$ be distinct nonzero homomorphisms from a field K into a field L. Show that the $\sigma_i{}'s$ are linearly independent over L, i.e. that if $a_1, a_2, \cdots, a_n \in L$ and $a_1\sigma_1(x) + a_2\sigma_2(x) + \cdots + a_n\sigma_n(x) = 0$ for all $x \in K$, then $a_1 = a_2 = \cdots = a_n = 0$.
6. Show that there is no automorphism of R other than the identity mapping.
7. Let V be a vector space of dimension 20. If V_1, V_2, V_3, V_4 are subspaces of dimension 9, 12, 10, 13 respectively, and $W = (V_1 \cap V_2) + (V_3 \cap V_4)$,
    1. find max{dim W} and the conditions for the max to be attained.
    2. same for min{dim W}.
8. Find the rational canonical form of the matrix

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 \end{bmatrix}$$

(i) over $\mathbb{Q}$, (ii) over $\mathbb{R}$, (iii) over $\mathbb{Z}/(17)$.