9th homework  
Due date: 5/19

Exercise 1. Let  
\[ A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & -2 & 1 \\ 1 & 3 & -2 \end{pmatrix} \in M_3(\mathbb{R}). \]

Let \( b = (3, -2, 1)^t \in \mathbb{R}^3 \). Find the minimum  
\[ \min_{x \in \mathbb{R}^3} \|Ax - b\|. \]

For any matrix \( A \in M_n(\mathbb{R}) \), denote by \( \rho_A \) the spectral radius of \( A \).

Exercise 2. Let \( A \in M_n(\mathbb{R}) \) be an irreducible matrix. Suppose that \( u \in (\mathbb{R}_{\geq 0})^n \) with  
\[ Au - \rho_A u \in (\mathbb{R}_{\geq 0})^n. \]

Prove that \( Au = \rho_A u \).

(Hint: consider \( w = (A + I_n)^{n-1} u \) and show that \( Aw - \rho_A w \in (\mathbb{R}_{\geq 0})^n \).)

Exercise 3. Let \( A \in M_n(\mathbb{R}) \) be an irreducible matrix. Show that if \( u \in (\mathbb{R}_{\geq 0})^n \) is an eigenvector of \( A \), then \( u \in (\mathbb{R}_{>0})^n \).

Exercise 4. Let \( A = (a_{ij}) \in M_n(\mathbb{R}) \) be an irreducible matrix such that \( a_{11} > 0 \). Show that \( A \) is regular.

Exercise 5. Consider the following miniature web:

\[ A \]
\[ B \quad C \quad D \]
\[ \quad E \]

(1) Take the damping factor \( \alpha = 0.9 \). Find the Google matrix  
\[ G = \alpha P + \frac{1 - \alpha}{5} \cdot E \in M_5(\mathbb{R}) \]
associated with the above web, where \( P \) is the transition matrix of the web and \( E = e^t e \) with \( e = (1, 1, 1, 1, 1) \).

(2) Compute the Perron vector of \( G \). You may need to use matrix calculator for this problem.