5. Differentiating the equation implicitly with respect to \( x \),
\[
3x^2 + 3y^2 \frac{dy}{dx} = 0.
\]
So
\[
\frac{dy}{dx} = -\frac{x^2}{y^2}.
\]

25. Differentiating the equation implicitly with respect to \( x \),
\[
2x + y + x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0
\]
\[
\Rightarrow \frac{dy}{dx} = -\frac{(2x + y)}{x + 2y}.
\]
So
\[
\frac{dy}{dx}\bigg|_{(1,1)} = -\frac{3}{3} = -1.
\]
Hence the tangent line is
\[
(y - 1) = -(x - 1) \iff y = -x + 2.
\]

26. Differentiating the equation implicitly with respect to \( x \),
\[
2x + 2y + 2x \frac{dy}{dx} - 2y \frac{dy}{dx} + 1 = 0
\]
\[
\Rightarrow \frac{dy}{dx} = -\frac{(2x + 2y + 1)}{2x - 2y}.
\]
So
\[
\frac{dy}{dx}\bigg|_{(1,2)} = -\frac{7}{2} \bigg|_{(1,2)} = \frac{7}{2}.
\]
Hence the tangent line is
\[
(y - 2) = \frac{7}{2}(x - 1) \iff y = \frac{7}{2}x - \frac{3}{2}.
\]

27. Differentiating the equation implicitly with respect to \( x \),
\[
2x + 2y \frac{dy}{dx} = 2(2x^2 + 2y^2 - x)(4x + 4y \frac{dy}{dx} - 1).
\]
Take \( (x, y) = (0, \frac{1}{2}) \) into this equation,
\[
\frac{dy}{dx}\bigg|_{(0,\frac{1}{2})} = 1.
\]
Hence the tangent line is
\[
(y - \frac{1}{2}) = x \iff y = x + \frac{1}{2}.
\]
28. Differentiating the equation implicitly with respect to \( x \),
\[
\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}} \frac{dy}{dx} = 0
\]
\[
\Rightarrow \frac{dy}{dx} = -\frac{y^{-\frac{1}{3}}}{x^{-\frac{1}{3}}}.
\]
So
\[
\frac{dy}{dx} \bigg|_{(-3\sqrt{3},1)} = \frac{1}{\sqrt{3}}.
\]
Hence the tangent line is
\[
(y - 1) = \frac{1}{\sqrt{3}}(x + 3\sqrt{3}) \Leftrightarrow y = \frac{1}{\sqrt{3}}x + 4.
\]

29. Differentiating the equation implicitly with respect to \( x \),
\[
4(x^2 + y^2)(2x + 2y \frac{dy}{dx}) = 25(2x - 2y \frac{dy}{dx})
\]
Take \((x, y) = (3, 1)\) into this equation,
\[
\frac{dy}{dx} \bigg|_{(3,1)} = -\frac{9}{13}.
\]
Hence the tangent line is
\[
(y - 1) = -\frac{9}{13}(x - 3) \Leftrightarrow y = -\frac{9}{13}x + \frac{40}{13}.
\]

30. Differentiating the equation implicitly with respect to \( x \),
\[
2y \frac{dy}{dx}(y^2 - 4) + y^2(2y) \frac{dy}{dx} = 2x(x^2 - 5) + x^2(2x)
\]
Take \((x, y) = (0, -2)\) into this equation,
\[
\frac{dy}{dx} \bigg|_{(0,-2)} = 0.
\]
Hence the tangent line is
\[
y + 2 = 0 \Leftrightarrow y = -2.
\]

59. \( x^2 + y^2 = r^2 \) is a circle with center \( O \), and \( ax + by = 0 \) is a line through center \( O \) (assume \( a^2 + b^2 \neq 0 \)).
\[
x^2 + y^2 = r^2 \Rightarrow 2x + 2y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y}.
\]
For a point \( P_0(x_0, y_0) \),
\[
\frac{dy}{dx} \bigg|_{(x_0,y_0)} = -\frac{x_0}{y_0},
\]
and the slope of the line \( OP_0 \) is \( \frac{y_0}{x_0} \). Hence, the curves are orthogonal, and the families of curves are orthogonal trajectories of each other.
63. Take $y = 0$ into this equation, $x = \pm \sqrt{3}$.

$$x^2 - xy + y^2 = 3 \Rightarrow 2x - y - x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{2x - y}{x - 2y}$$

so

$$\left. \frac{dy}{dx} \right|_{(\sqrt{3}, 0)} = \left. \frac{dy}{dx} \right|_{(-\sqrt{3}, 0)} = 2.$$

The two tangent lines are

$$y = 2(x - \sqrt{3}), y = 2(x + \sqrt{3})$$

which are parallel.

69. Differentiating the equation implicitly with respect to $x$,

$$2x + 8y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-x}{4y}.$$ 

Let $h$ be the height of the lamp and $(a, b)$ be the intersection point of tangent and the ellipse. The tangent line has slope

$$\frac{h}{8} = \frac{a}{4b} = \frac{b}{a + 5}$$

By

$$a^2 + 4b^2 = 5,$$

we can compute

$$a = -1, b = 1, h = 2.$$