Advanced Algebra II
Homework 2
due on Mar. 12, 2004

(1) Let \( L_1, L_2 \) be two algebraically closed fields over \( K \). Show there is an \( K \)-embedding either from \( L_1 \) to \( L_2 \) or from \( L_2 \) to \( L_1 \).

(2) A module is said to be cyclic if it’s generated by one element. Show that the cyclic submodule generated by \( a \) is \( Ra + Za \). And in case \( M \) is unitary, then the cyclic submodule generated by \( a \) is \( Ra \).

(3) Give an example of finitely generated \( R \)-module which is not a finitely generated abelian group.

(4) Let \( f : M \rightarrow M \) such that \( f \circ f = f \), then \( M = \text{Ker}f \oplus \text{Im}f \).

(5) Let \( f : M \rightarrow N \) and \( g : N \circ M \) be \( R \)-homomorphism such that \( gf = 1_M \), then \( N = \text{Im}f \oplus \text{Ker}g \).