Advanced Algebra I
Homework 7
due on Nov.7, 2003

Part A.

(1) Let \((V, \rho)\) be a the induced representation of \((W, \theta)\). If \(C\) is the conjugacy class of \(u\), and \(C \cap H\) decompose into conjugacy classes \(D_1, \ldots, D_r\) of \(H\). Then

\[
\chi_{\rho}(u) = \frac{|G|}{|H|} \sum_{i=1}^{r} \frac{|D_i|}{|C|} \chi_{\theta}(D_i).
\]

(2) Let \(H = \langle (1234) \rangle\) be a subgroup of \(S_4\). Determine the isomorphic classes of representations of \(S_4\) induced by irreducible representations of \(H\).

(3) Which irreducible representation of \(S_4\) remain irreducible when restricted to \(A_4\)? Which are induced from \(A_4\)?

Part B.

(1) Determine the character table for \(S_5\) and \(A_5\).

(2) Show that \(G\) is simple if and only if every non-trivial character has kernel \(\{1\}\).