Advanced Algebra I
Homework 2
due on Oct.3, 2003

Part A.
(1) Find the Sylow 2-subgroup and 3-subgroup of $S_4, S_6$
(2) Let $G$ be a group. Show that $Aut(G)$, the set of automorphism, is a group by composition. Furthermore, consider the conjugation $G \times G \rightarrow G$, we have and group homomorphism $\tilde{\alpha} : G \rightarrow Aut(G)$. What is kernel of $\tilde{\alpha}$?
(3) Show that group of order $p^2$ is abelian.
(4) Show that $D_6$ is not isomorphic to $A_4$. (They both are non-abelian group of order 12).

Part B.
(1) Show that $Z(S_n) = \{e\}$. And show that $Aut(S_4) \cong S_4$.
(2) Let $p$ be a odd prime. Prove that there are at most two non-abelian group of order $p^3$. 