
Solutions to exercises in Section 5.6: #10, #12, #64, #75, #76, #77, #78.

#10. Since $xy + y' = 100x$, we have $y' = 100x - xy = x(100 - y)$ or $\frac{y'}{100 - y} = x$. Then

$$\int \frac{y'}{100 - y} \, dx = \int x \, dx \quad \text{or} \quad \int \frac{1}{100 - y} \, dy = \int x \, dx,$$

which gives $-\ln(100 - y) = x^2/2 + C_1$ or $100 - y = e^{-x^2/2}e^{-C_1}$ or $y = 100 - Ce^{-x^2/2}$.

#12. The differential equation is $\frac{dP}{dt} = k(10 - t)$. Then

$$\int \frac{dP}{dt} \, dt = \int k(10 - t) \, dt \quad \text{and so} \quad P = -k(10 - t)^2/2 + C.$$

#64. The relation between the annual revenue and time is $y = Ce^{kt}$, with the initial conditions that $y = 742000$ for $t = 0$ and $y = 632000$ for $t = 2$. Then $742000 = Ce^0 = C$ and $632000 = Ce^{2k}$; which gives $C = 742000$ and $e^{2k} = 632000/742000$. Then the revenue for 2002 is $Ce^{4k} = C(e^{2k})^2 = 742000(632000/742000)^2 \approx 538372$.

#75. False. If $y = Ce^{kt}$, then $y' = Cke^{kt}$ is not a constant.

#76. True. If $y = ax + b$, then $y' = a$ is a constant.

#77. False. The rising rate for a year is $(1 + 5\%)^{12} - 1 \approx 6.17\%$.

#78. True.