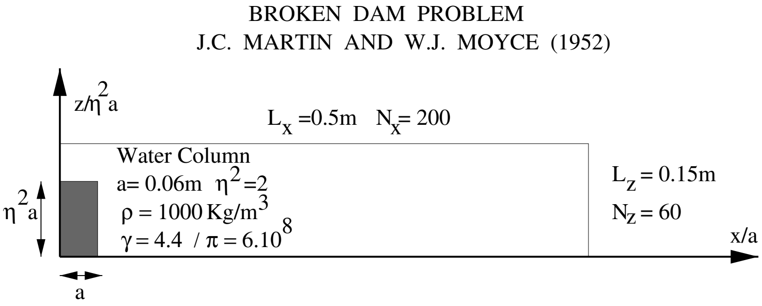

Fig. 15. Initial configuration for the broken dam problem.

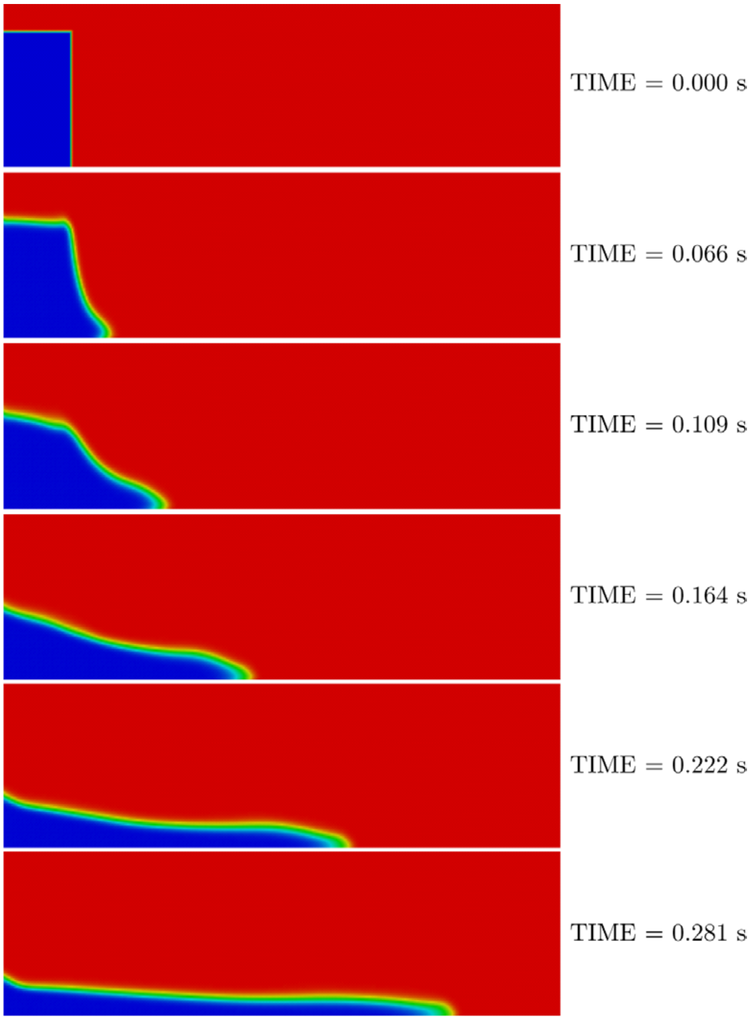

Fig. 16. Isovalues of the volume fraction for the broken dam problem.

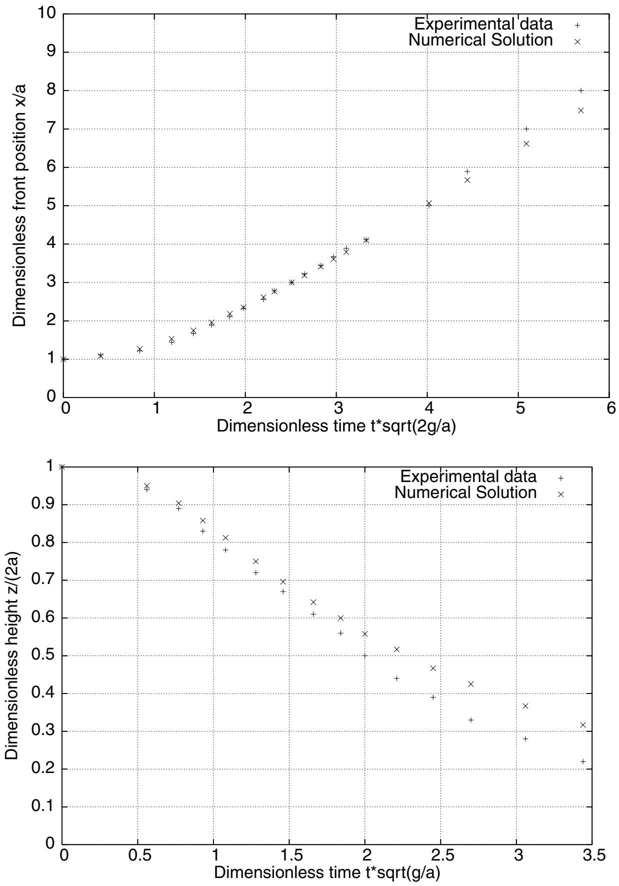

Fig. 17. Comparison between numerical solution and experimental results for the broken dam problem. Front position (top) and height of the column (bottom).

totally identical with those obtained in [12], a good agreement with these results is reached. Again, we emphasize that in the limit where the space step goes to zero, for this interface problems between compressible fluids, the two models have to give exactly the same results. For this computation, a mesh of 400×400 thus seems sufficient for this purpose.

5.3.3. The broken dam problem

Finally, we present here a computation of the well known broken dam problem of Martin and Moyce [11]. This test consists of the simple configuration represented in Fig. 15. Initially a water column with $a = 0.06$ m wide and $\eta^2 a = 0.12$ m high is a rest. All the boundaries are solid walls. Under the effect of the gravity $g = 9.81$ m s^{-2}, the column collapses. The computation is made with the acoustic solver. The mesh contains 200×60 nodes and the CFL number is equal to 0.8. The experimental results of [11], for the front position $x/a = F_1(\eta^2, t\sqrt{2g/a})$ and the height of the column $z/(\eta^2 a) = F_2(\eta^2, t\sqrt{g/a})$ are used for comparison (note that in the experimental results, the non-dimensional times are different for the front position and the water height: we have used the same non-dimensional units in the presentation of the numerical results to allow a direct comparison with the original publication of [11]). In Fig. 17, are compared the numerical results and the experimental ones for the case $\eta^2 = 2$. Although, the Mach number is very small, we note a good agreement with the experimental results. Fig. 16 shows the isovalues of the volume fraction at the different dimensionless time $t\sqrt{2g/a} = 0, 1.19, 1.98, 2.97, 4.02, 5.09$ corresponding to the physical times $t = 0$, 0.066, 0.109, 0.164, 0.222, 0.281 s.

6. Conclusion

We have derived a five equation reduced model from an asymptotic analysis in the limit of zero relaxation time of a seven equation two velocity, two pressure model. Although, this model cannot be cast in conservative form, the mathematical structure of the model have been analyzed and shown to be very close to the structure of the Euler equations of fluid dynamics. This model presents an interesting alternative to the use of the seven equation model: it is cheaper, simpler to implement and is easily extensible to an arbitrary number of materials. For instance, in three dimensions, for a number k of different material, the two velocity, two pressure model uses $6k - 1$ variables while the reduced model will use only $2k + 3$ variables.

From a numerical point of view, we have proposed two different approximation schemes of this system. The first one (VFRoe-ncv) relies on an approximate linearized Riemann solver. The second one, that is useful for the simulation of interface problems in low Mach number flows, uses the mathematical structure of the model and relies on the linearization of characteristic relations.

The numerical results show that the reduced five equation model is able of accurate computations of interface problems between compressible material as well as of some two-phase flow problems where pressure and velocity equilibrium between the phases is reached. The numerical methods have been shown to be efficient and robust for a large range of Mach number from almost zero to 1.9 and for density ratio as large as 1000. In the future, we plan to improve the efficiency and accuracy of the numerical method for low Mach number flows by designing implicit and preconditioned schemes for this model. Preliminary results in this direction are promising [13].

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