Wave Propagation Methods for Compressible Multicomponent Flow with Moving Interfaces and Boundaries

Keh-Ming Shyue

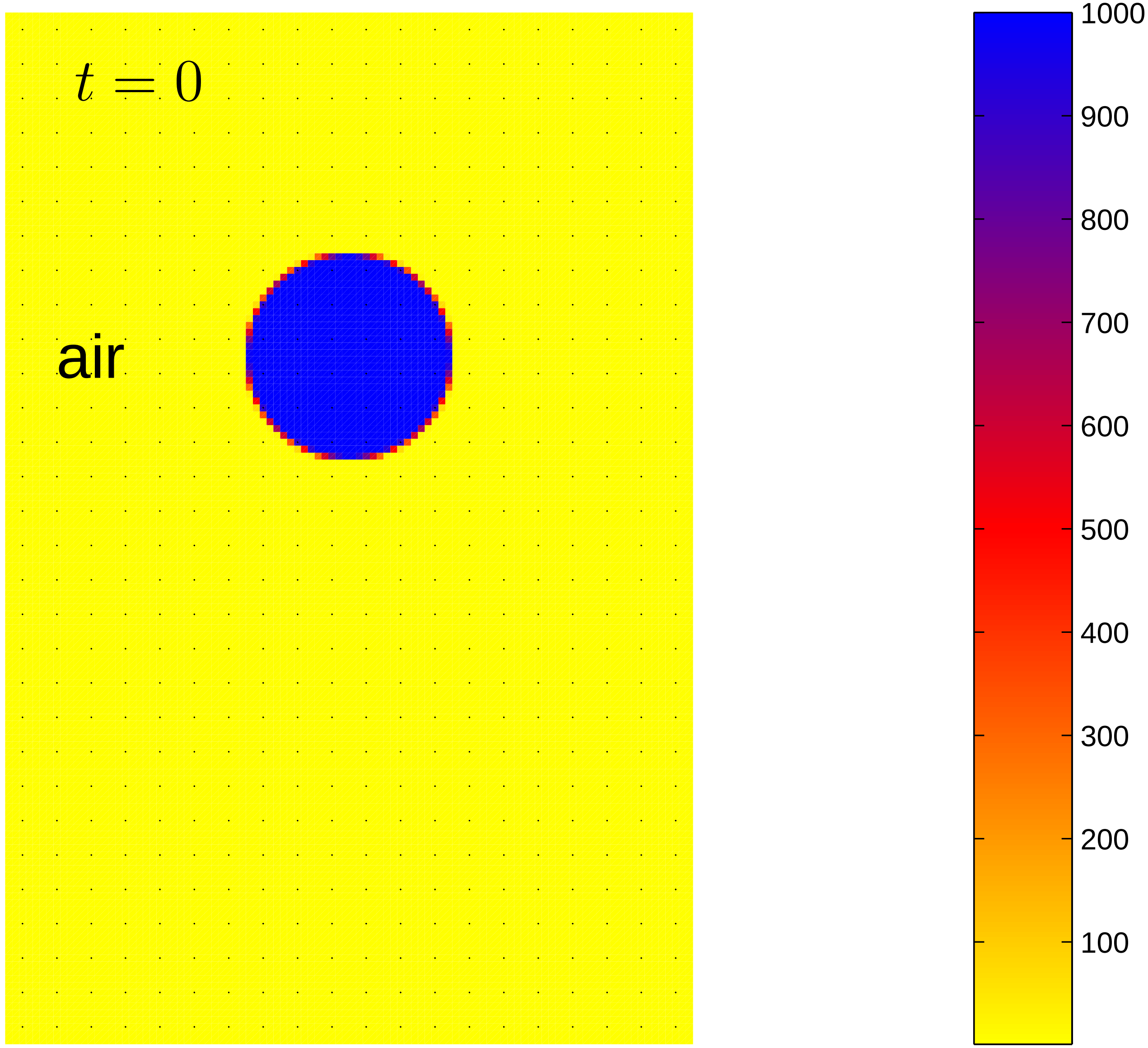
Department of Mathematics

National Taiwan University

Overview

- Illustrative examples
- Mathematical model
  - Fluid-mixture type equations of motion for homogeneous two-phase flow
  - General pressure law for real materials
- Numerical techniques
  - Finite volume method based on wave propagation
    - Surface tracking for moving boundaries
    - Volume tracking for moving interfaces
- Numerical results
- Future work

Falling Liquid Drop Problem



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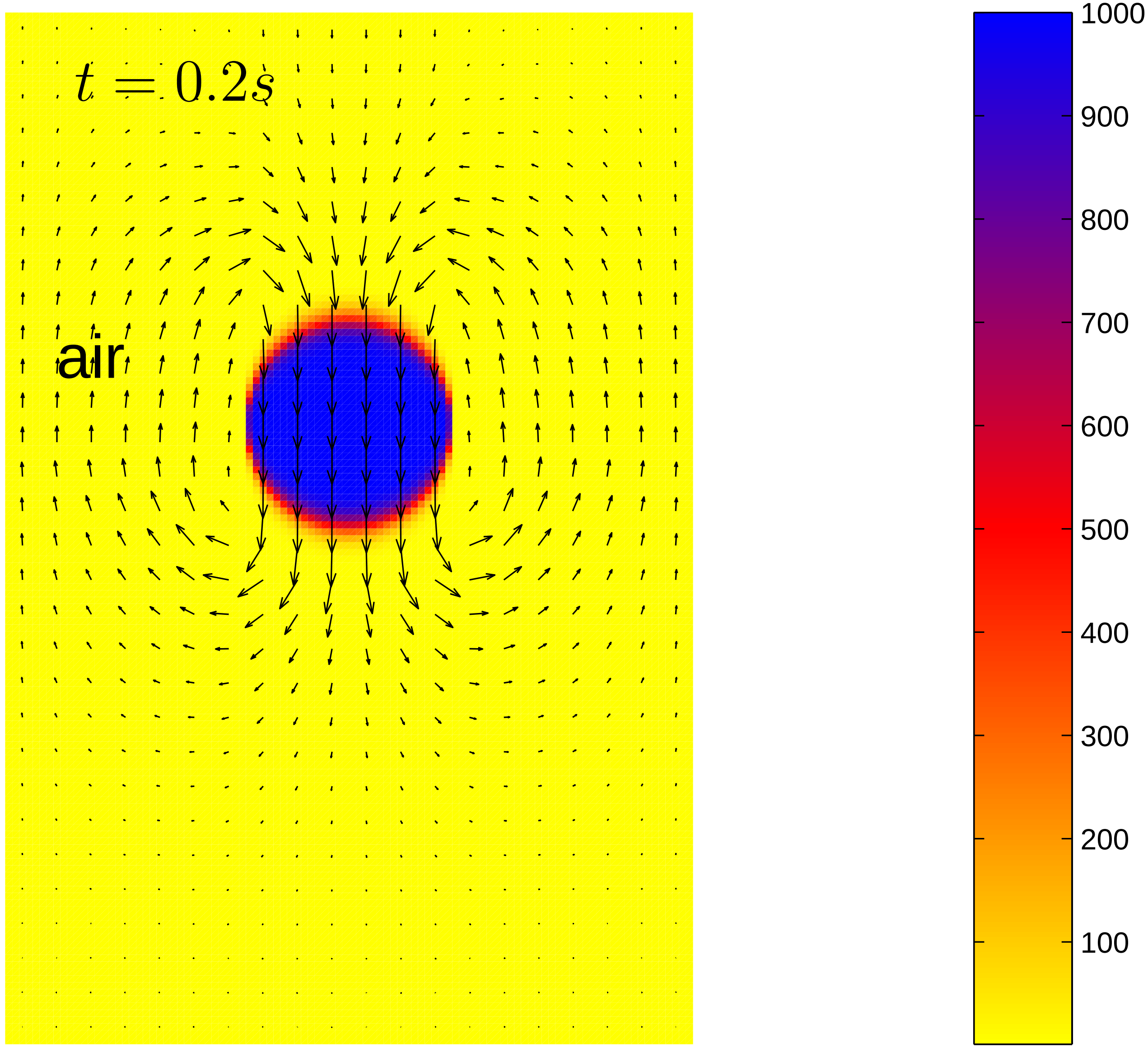

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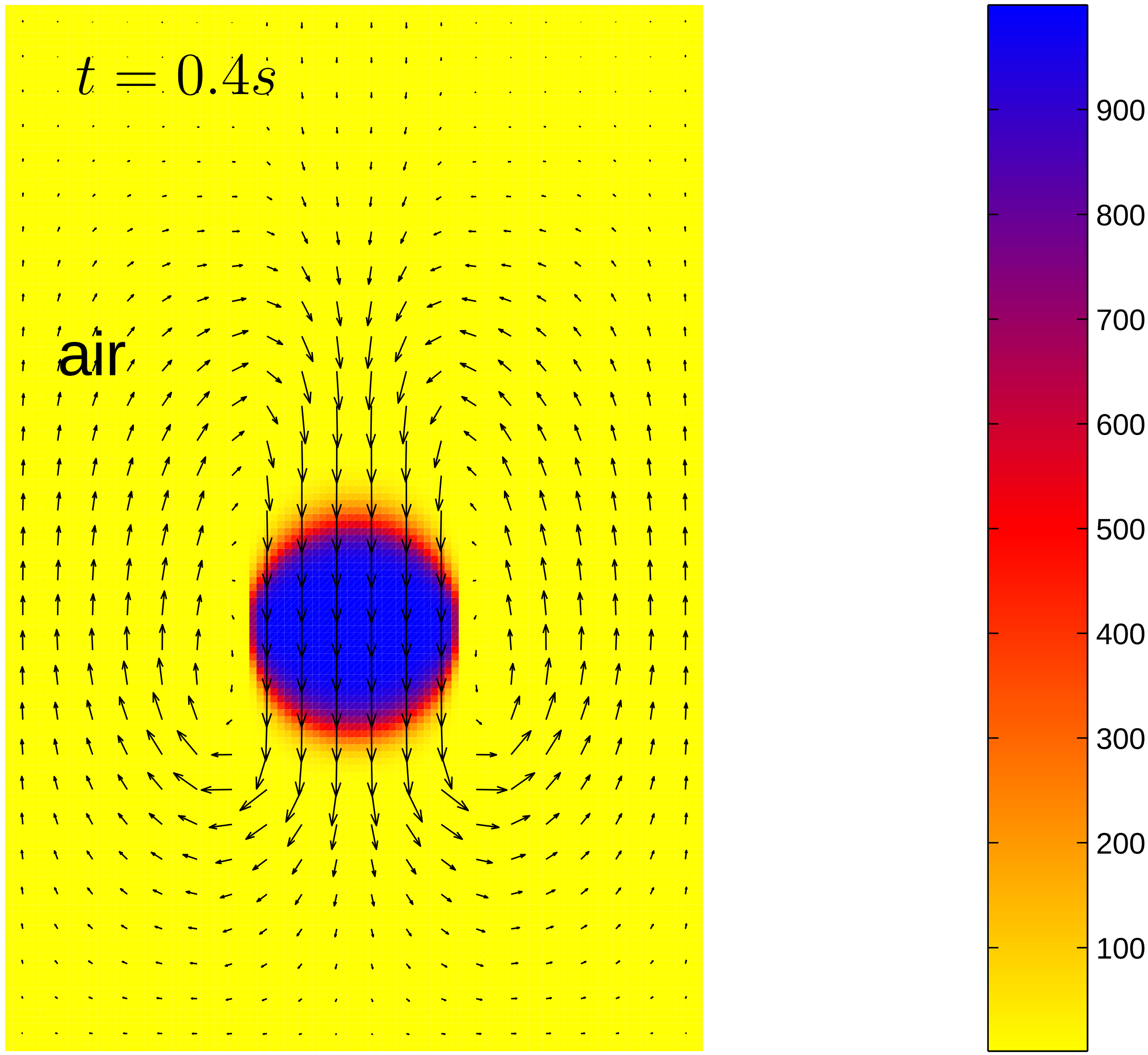

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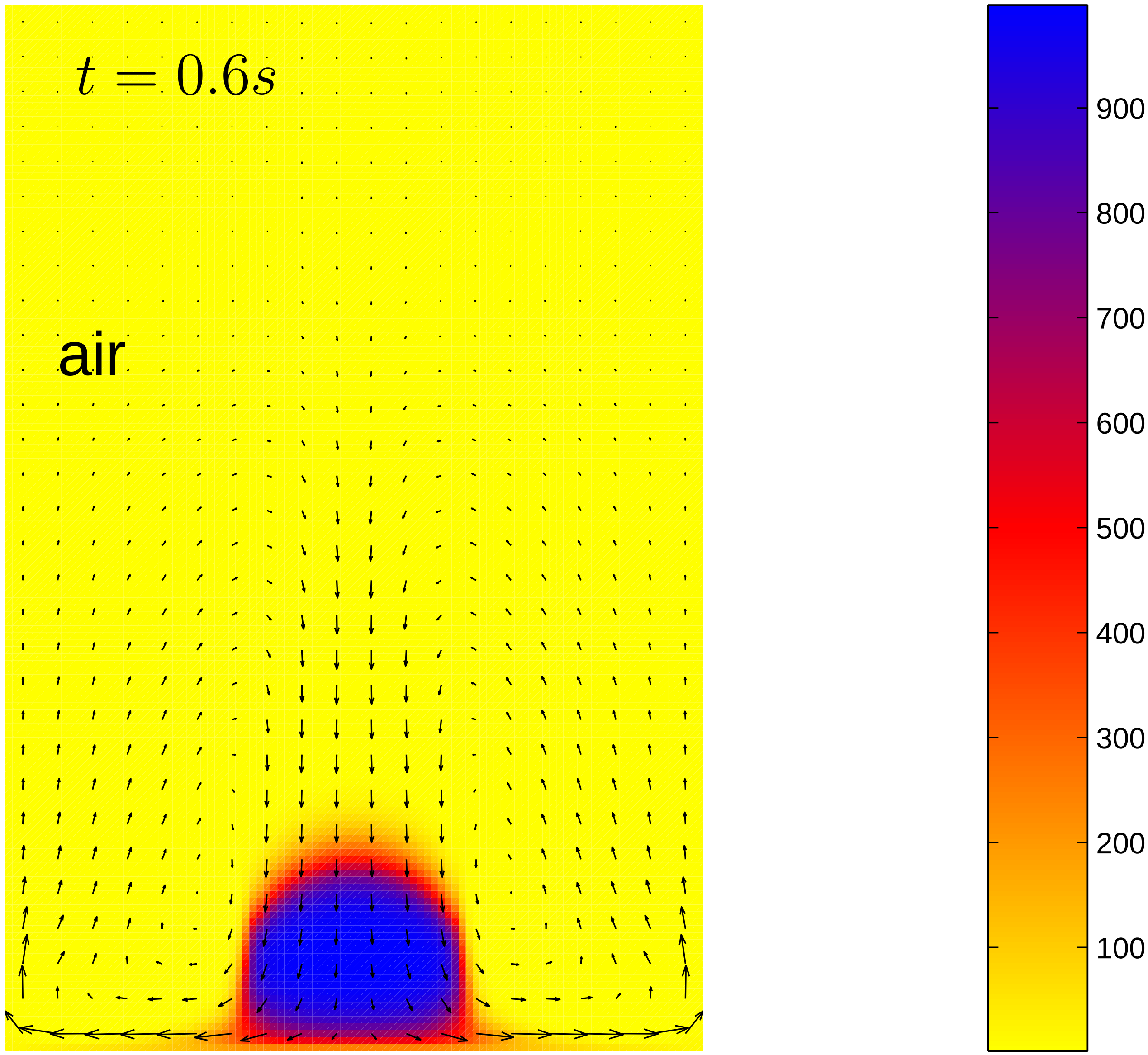

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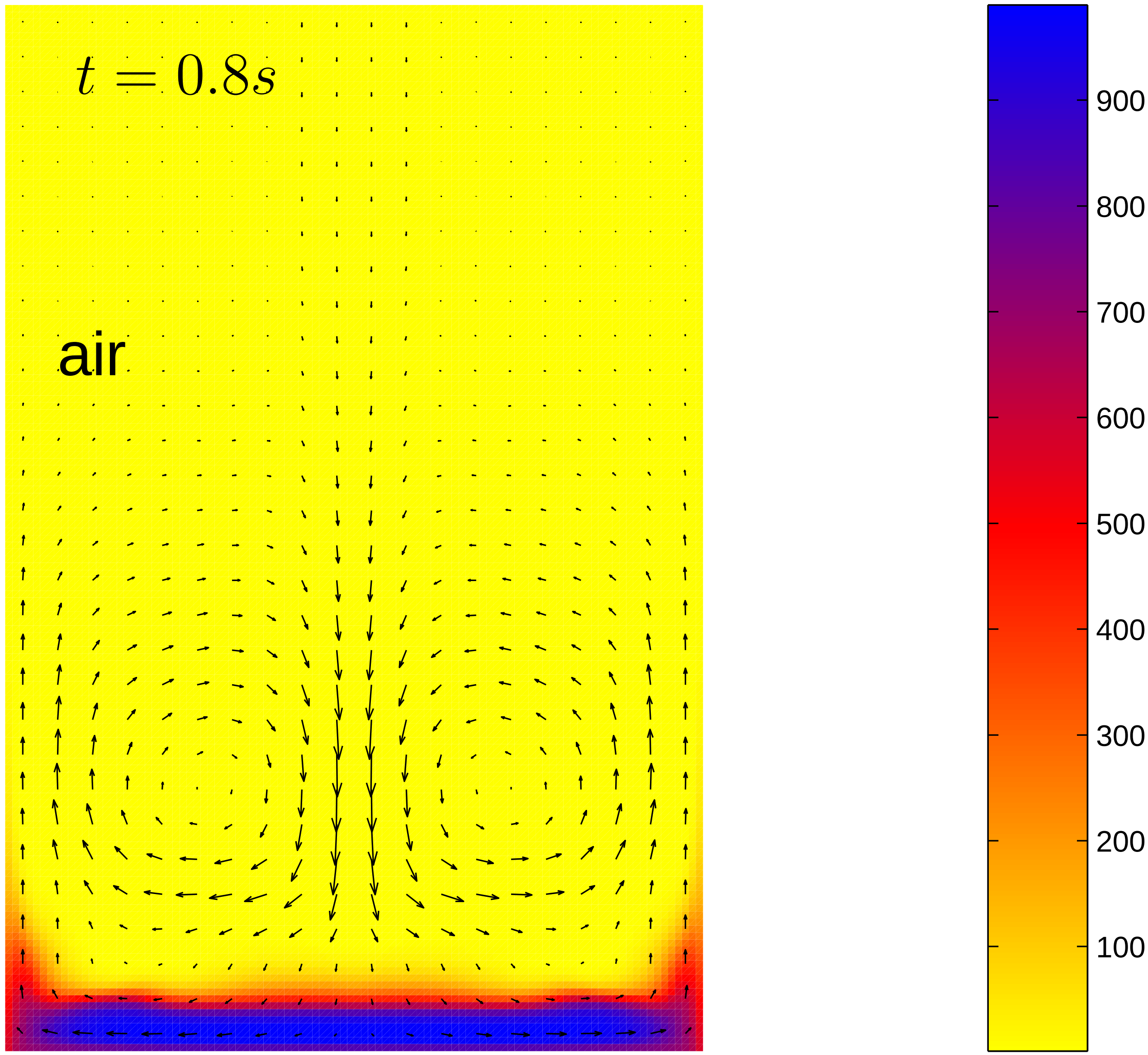

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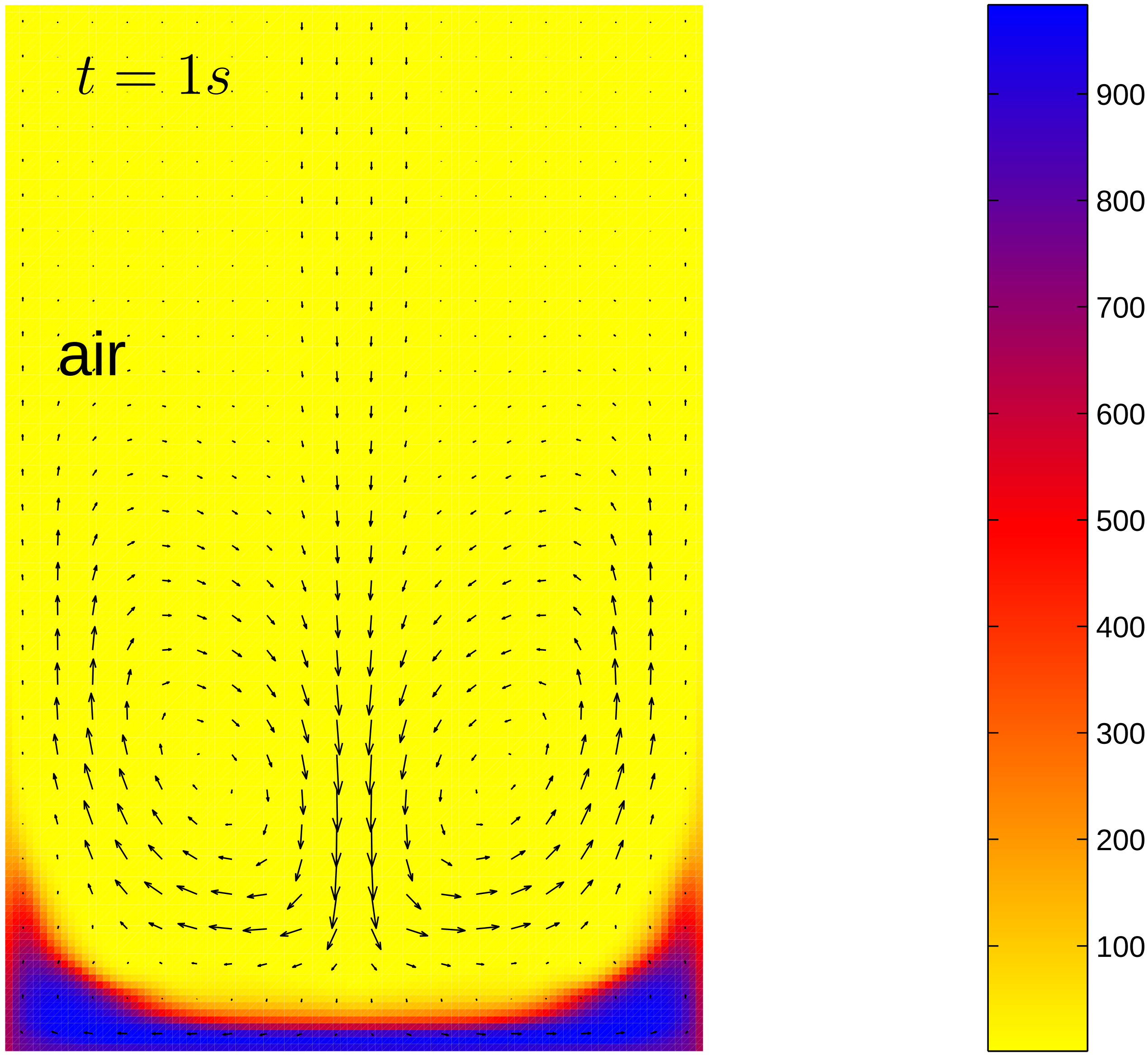
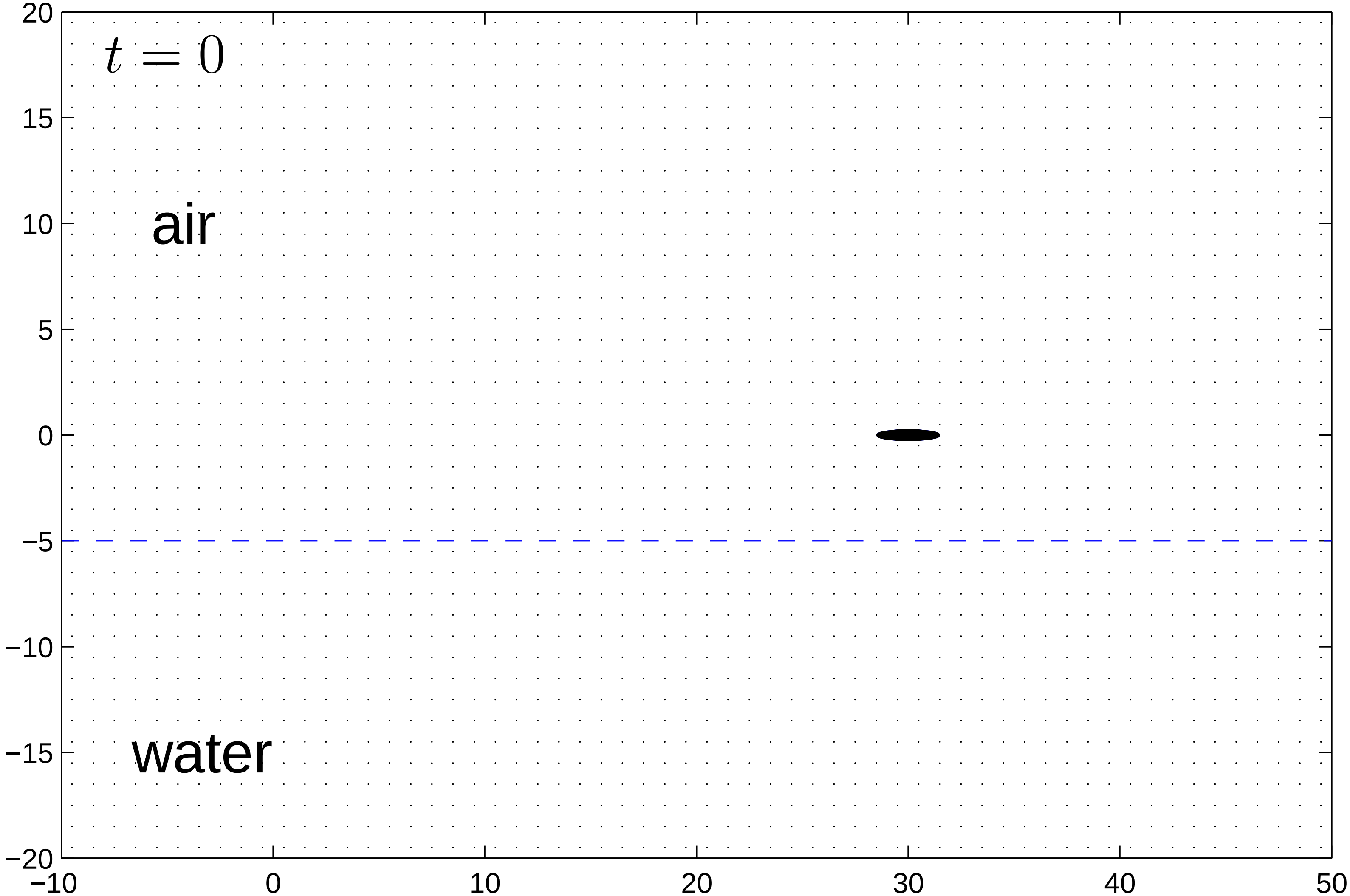


Flying Projectile & Ocean Surface



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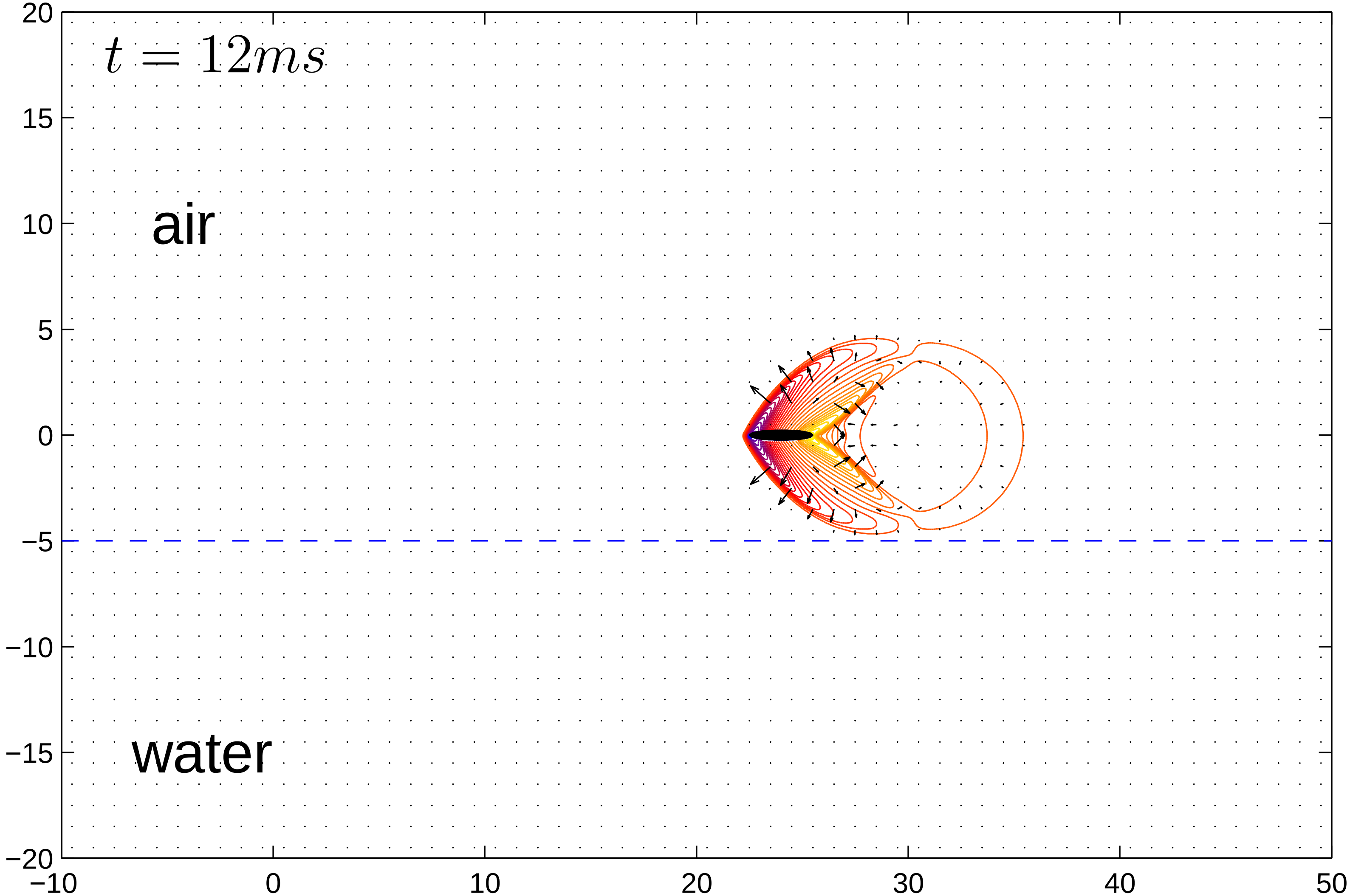

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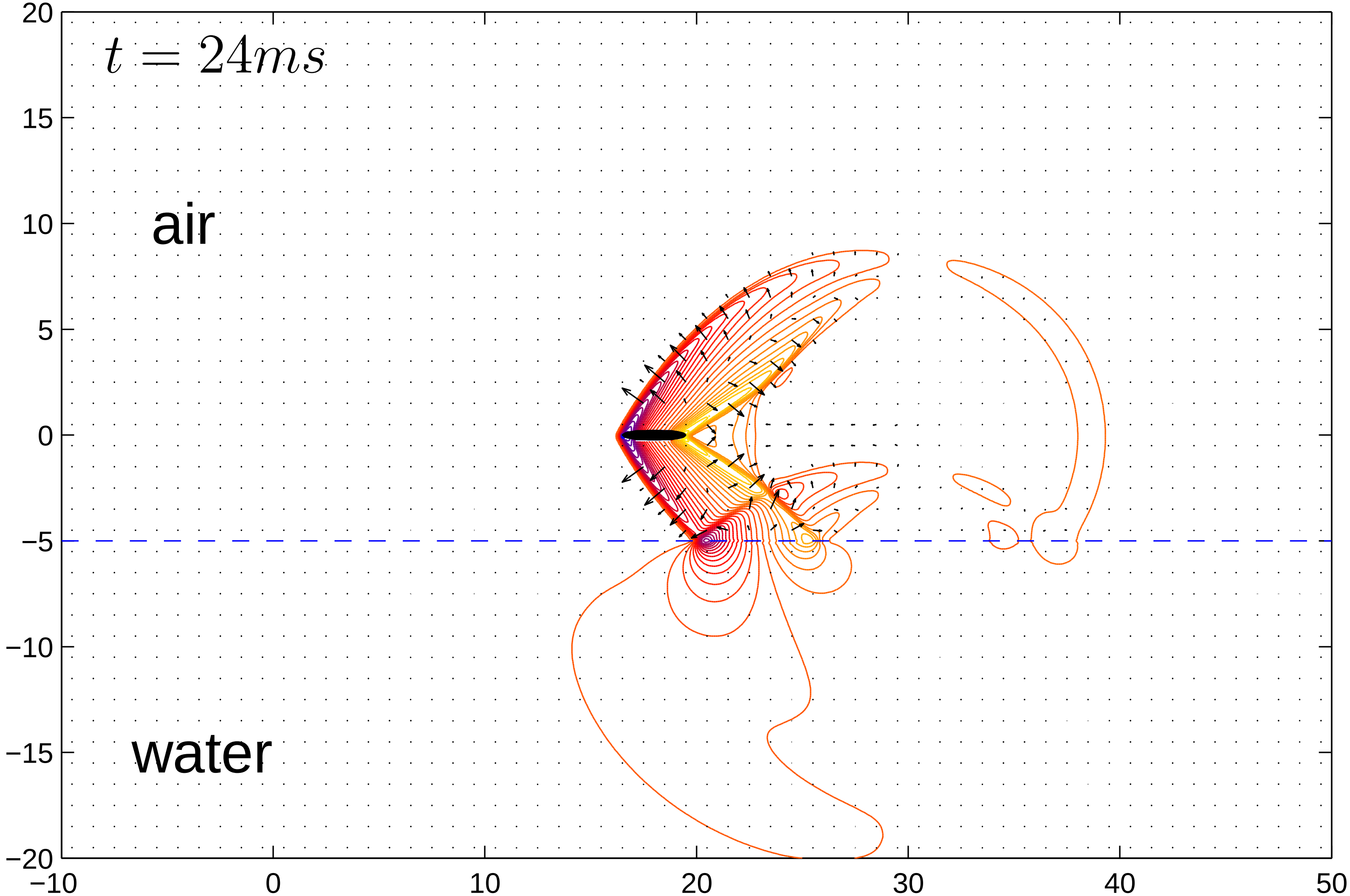

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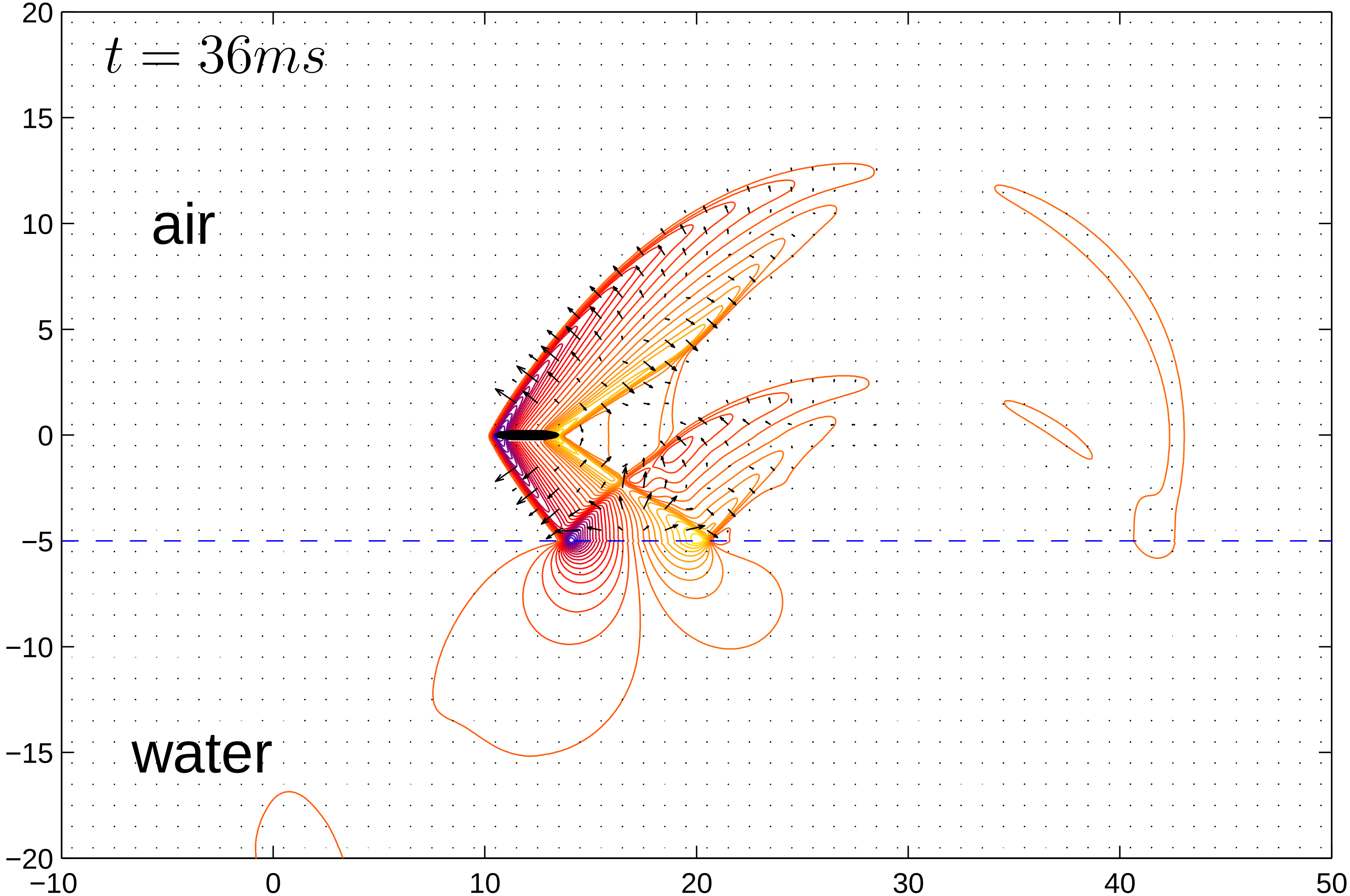

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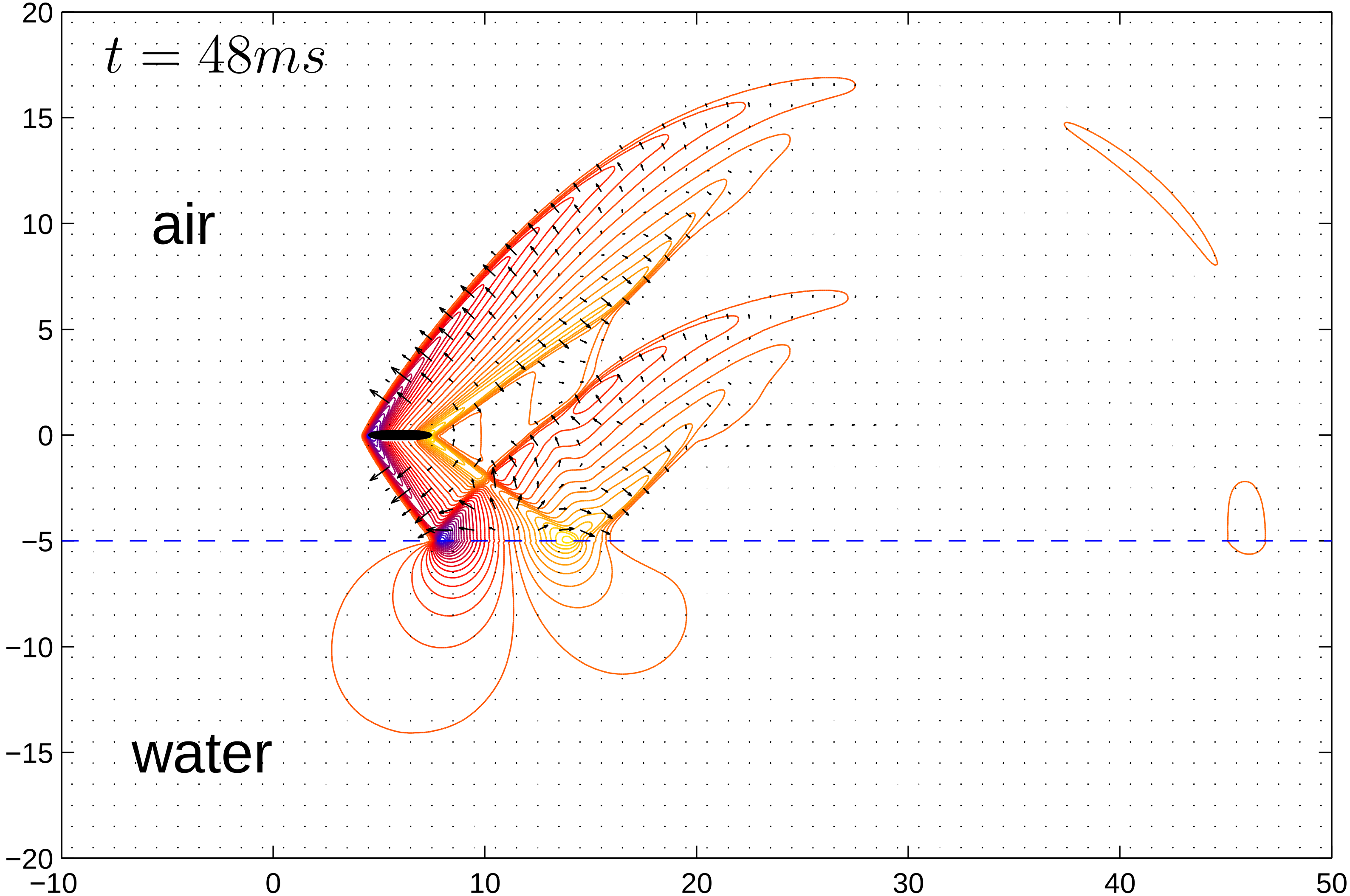

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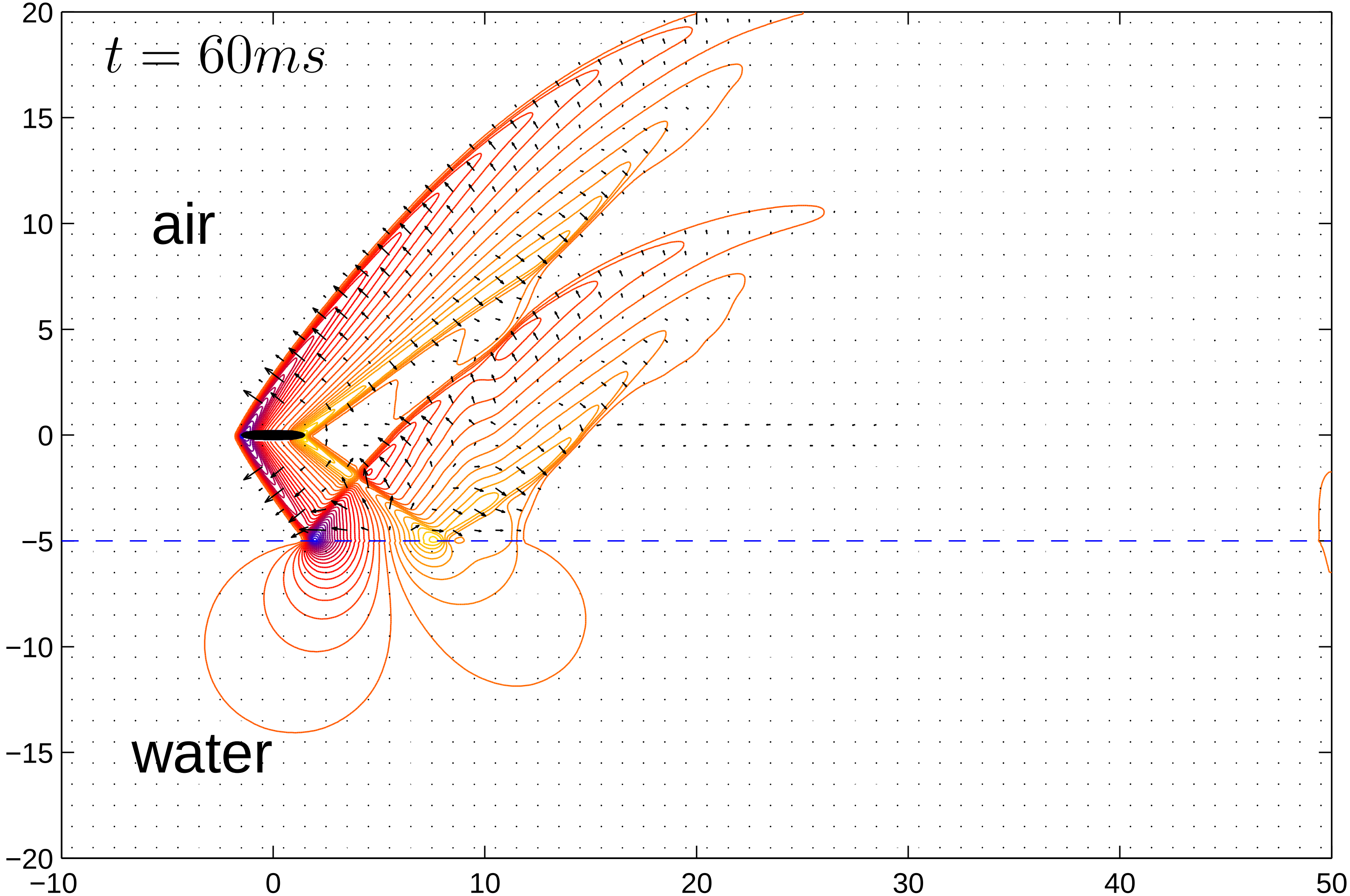

Two Phase Flow Problem

Ignore physical effects such as gravity, viscosity, surface tension, mass diffusion, and so on

Each fluid component satisties

- Eulerian form conservation laws

$$
\begin{aligned}
\rho_t + \nabla \cdot (\rho \vec{u}) &= 0 \\
(\rho \vec{u})_t + \nabla \cdot (\rho \vec{u} \otimes \vec{u}) + \nabla p &= 0 \\
(\rho E)_t + \nabla \cdot (\rho E \vec{u} + p \vec{u}) &= 0
\end{aligned}
$$

- General pressure law $p(\rho, e)$

ρ: density, $\vec{u}$: vector of particle velocity, p: pressure
E: specific total energy, e: specific internal energy

Two-Phase Flow Model

- Model derivation based on averaging theory of Drew (Theory of Multicomponent Fluids, D.A. Drew & S. L. Passman, Springer, 1999)

Namely, introduce indicator function χ_k as

$$\chi_k(M,t) = \begin{cases} 1 & \text{if } M \text{ belongs to phase } k \\ 0 & \text{otherwise} \end{cases}$$

Denote $<\psi>$ as volume averaged for flow variable ψ,

$$\langle\psi\rangle = \frac{1}{V}\int_V \psi \, dV$$

Gauss & Leibnitz rules

$$\langle\chi_k\nabla\psi\rangle = \langle\nabla(\chi_k\psi)\rangle - \langle\psi\nabla\chi_k\rangle \quad \& \quad \langle\chi_k\psi_t\rangle = \langle(\chi_k\psi)_t\rangle - \langle\psi(\chi_k)_t\rangle$$

Two-Phase Flow Model (cont.)

Take product of each conservation law with χ_k & perform averaging process. In case of mass conservation equation, for example, we have

$$\langle \chi_k \rho_k \rangle_t + \nabla \cdot < \chi_k \rho_k \vec{u}_k >= \langle \rho_k (\chi_k)_t + \rho_k \vec{u}_k \cdot \nabla \chi_k \rangle$$

Since χ_k is governed by

$$(\chi_k)_t + \vec{u}_0 \cdot \nabla \chi_k = 0 \qquad (\vec{u}_0\text{: interface velocity}),$$

this leads to mass averaged equation for phase k

$$\langle \chi_k \rho_k \rangle_t + \nabla \cdot < \chi_k \rho_k \vec{u}_k >= \langle \rho_k \left(\vec{u}_k - \vec{u}_0 \right) \cdot \nabla \chi_k \rangle$$

Analogously, we may derive averaged equation for momentum, energy, & entropy (not shown here)

Two-Phase Flow Model (cont.)

In summary, averaged model system, we have, are

$$
\begin{aligned}
\langle \chi_k \rho_k \rangle_t + \nabla \cdot < \chi_k \rho_k \vec{u}_k >= & \langle \rho_k \left(\vec{u}_k - \vec{u}_0 \right) \cdot \nabla \chi_k \rangle \\
\langle \chi_k \rho_k \vec{u}_k \rangle_t + \nabla \cdot < \chi_k \rho_k \vec{u}_k \otimes \vec{u}_k > + \nabla \langle \chi_k p_k \rangle = & \langle p_k \nabla \chi_k \rangle + \\
& \langle \rho_k \vec{u}_k \left(\vec{u}_k - \vec{u}_0 \right) \cdot \nabla \chi_k \rangle \\
\langle \chi_k \rho_k E_k \rangle_t + \nabla \cdot < \chi_k \rho_k E_k \vec{u}_k + \chi_k p_k \vec{u}_k >= & \langle p_k \vec{u}_k \cdot \nabla \chi_k \rangle + \\
& \langle \rho_k E \left(\vec{u}_k - \vec{u}_0 \right) \cdot \nabla \chi_k \rangle \\
\langle \chi_k \rangle_t + \langle \vec{u}_k \cdot \nabla \chi_k \rangle = & \langle \left(\vec{u}_k - \vec{u}_0 \right) \cdot \nabla \chi_k \rangle
\end{aligned}
$$

Note: existence of various interfacial source terms
Mathematical as well as numerical modelling of these terms are important (but difficult) for general multiphase flow problems

Homogeneous 2-Phase Flow Model

- Assume homogeneous (1-pressure & 1-velocity) flow; *i.e.*, across interfaces: $p_\iota = p$ & $\vec{u}_\iota = \vec{u}$, $\iota = 0, 1, 2$
- Introduce volume fraction for phase k as $\alpha_k = V_k / V$

Now, by dropping all interfacial terms, we may obtain a simplified model as

$$
\begin{aligned}
(\alpha_k \rho_k)_t + \nabla \cdot (\alpha_k \rho_k \vec{u}) &= 0 \\
(\alpha_k \rho_k \vec{u})_t + \nabla \cdot (\alpha_k \rho_k \vec{u} \otimes \vec{u}) + \nabla (\alpha_k p) &= p \nabla \alpha_k \\
(\alpha_k \rho_k E_k)_t + \nabla \cdot (\alpha_k \rho_k E_k \vec{u} + \alpha_k p \vec{u}) &= p \vec{u} \cdot \nabla \alpha_k \\
(\alpha_1)_t + \vec{u} \cdot \nabla \alpha_1 &= 0
\end{aligned}
$$

for $k = 1, 2$, & $\alpha_1 + \alpha_2 = 1$. Note this gives $2(2 + N_d) + 1$ equations in total for a N_d-dimension 2-phase flow problem

Homogeneous Flow Model (cont.)

Note that, in practice, rather than using equations $\alpha_k \rho_k \vec{u}$ & $\alpha_k \rho_k E_k$ for each phase, we may write down a system of the form

$$
\begin{aligned}
(\alpha_k \rho_k)_t + \nabla \cdot (\alpha_k \rho_k \vec{u}) &= 0 \\
(\rho \vec{u})_t + \nabla \cdot (\rho \vec{u} \otimes \vec{u}) + \nabla p &= 0 \\
(\rho E)_t + \nabla \cdot (\rho E \vec{u} + p \vec{u}) &= 0 \\
(\alpha_1)_t + \vec{u} \cdot \nabla \alpha_1 &= 0
\end{aligned}
$$

$\rho \vec{u} = \sum_{k=1}^{2} \alpha_k \rho_k \vec{u}$: total momentum
$\rho E = \sum_{k=1}^{2} \alpha_k \rho_k E_k$: total energy

This gives $4 + N_d$ equations in total, $N_d + 1$ less than previous model system

Homogeneous Flow Model (cont.)

Note that it is easy to include, for instance, gravity & capillary effects to the model

$$
\begin{aligned}
(\alpha_k \rho_k)_t + \nabla \cdot (\alpha_k \rho_k \vec{u}) &= 0 \qquad (k = 1, 2) \\
(\rho \vec{u})_t + \nabla \cdot (\rho \vec{u} \otimes \vec{u}) + \nabla p &= \vec{\phi} \\
(\rho E)_t + \nabla \cdot (\rho E \vec{u} + p \vec{u}) &= \vec{\phi} \cdot \vec{u} \\
(\alpha_1)_t + \vec{u} \cdot \nabla \alpha_1 &= 0
\end{aligned}
$$

1. Gravity case: $\vec{\phi} = \vec{g}$
2. Capillary case: $\vec{\phi} = \sigma \kappa \nabla \alpha$

$\vec{g}$: gravitational constant, σ: surface tension coef.
κ: curvature at interface

Homogeneous Flow Model (cont.)

- Mixture equation of state: $p = p(\alpha_2,\ \alpha_1\rho_1,\ \alpha_2\rho_2,\ \rho e)$
- Isobaric closure: $p_1 = p_2 = p$
  - For a class of EOS, explicit formula for p is available (examples are given next)
  - For some complex EOS, from $(\alpha_2, \rho_1, \rho_2, \rho e)$ in model equations we recover p by solving

$$p_1(\rho_1, \rho_1 e_1) = p_2(\rho_2, \rho_2 e_2) \quad \& \quad \sum_{k=1}^{2} \alpha_k \rho_k e_k = \rho e$$

- This homogeneous two-phase model was called a five-equation model by Allaire, Clerc, & Kokh (JCP 2002) or a volume-fraction model by Shyue (JCP 1998)

Homogeneous Flow Model (cont.)

- Polytropic ideal gas: $p_k = (\gamma_k - 1)\rho_k e_k$

$$\rho e = \sum_{k=1}^{2} \alpha_k \rho_k e_k = \sum_{k=1}^{2} \alpha_k \frac{p}{\gamma_k - 1} \quad \Longrightarrow$$

$$p = \rho e \Bigg/ \sum_{k=1}^{2} \frac{\alpha_k}{\gamma_k - 1}$$

Homogeneous Flow Model (cont.)

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$$p = \rho e \Big/ \sum_{k=1}^{2} \frac{\alpha_k}{\gamma_k - 1}$$

- Van der Waals gas: $p_k = \left(\frac{\gamma_k - 1}{1 - b_k \rho_k}\right)(\rho_k e_k + a_k \rho_k^2) - a_k \rho_k^2$

$$\rho e = \sum_{k=1}^{2} \alpha_k \rho_k e_k = \sum_{k=1}^{2} \alpha_k \left[\left(\frac{1 - b_k \rho_k}{\gamma_k - 1} \right) (p + a_k \rho_k^2) - a_k \rho_k^2 \right] \quad \Longrightarrow$$

$$p = \left[\rho e - \sum_{k=1}^{2} \alpha_k \left(\frac{1 - b_k \rho_k}{\gamma_k - 1} - 1 \right) a_k \rho_k^2 \right] \Big/ \sum_{k=1}^{2} \alpha_k \left(\frac{1 - b_k \rho_k}{\gamma_k - 1} \right)$$

Homogeneous Flow Model (cont.)

- Two-molecular vibrating gas: $p_k = \rho_k R_k T(e_k)$, T satisfies

$$e = \frac{RT}{\gamma - 1} + \frac{RT_{\text{vib}}}{\exp\left(T_{\text{vib}}/T\right) - 1}$$

As before, we now have

$$\begin{aligned}
\rho e = \sum_{k=1}^{2} \alpha_k \rho_k e_k &= \sum_{k=1}^{2} \alpha_k \left[\left(\frac{\rho_k R_k T_k}{\gamma_k - 1} \right) + \frac{\rho_k R_k T_{\text{vib},k}}{\exp\left(T_{\text{vib},k}/T_k\right) - 1} \right] \\
&= \sum_{k=1}^{2} \alpha_k \left[\left(\frac{p}{\gamma_k - 1} \right) + \frac{p_{\text{vib},k}}{\exp\left(p_{\text{vib},k}/p\right) - 1} \right] \quad \text{(Nonlinear eq.)}
\end{aligned}$$

Homogeneous Flow Model (cont.)

- It is easy to show entropies, $\mathcal{S}_k$, $k = 1, 2$, satisfy

$$\left(\frac{\partial p_1}{\partial \mathcal{S}_1}\right)_{\rho_1} \frac{D\mathcal{S}_1}{Dt} - \left(\frac{\partial p_2}{\partial \mathcal{S}_2}\right)_{\rho_2} \frac{D\mathcal{S}_2}{Dt} = \left(\rho_1 c_1^2 - \rho_2 c_2^2\right) \nabla \cdot \vec{u}$$

Homogeneous Flow Model (cont.)

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- Murrone & Guillard (JCP 2005) propsed a reduced two-phase flow model in which

$$(\alpha_1)_t + \vec{u} \cdot \nabla \alpha_1 = \alpha_1 \alpha_2 \left(\frac{\rho_2 c_2^2 - \rho_1 c_1^2}{\sum_{k=1}^{2} \alpha_k \rho_k c_k^2}\right)$$

and now entropy of each phase satisfy

$$\frac{D\mathcal{S}_k}{Dt} = \frac{\partial \mathcal{S}_k}{\partial t} + \vec{u} \cdot \nabla \mathcal{S}_k = 0, \quad \text{for} \quad k = 1, 2$$

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4. In fact, there are other set of model systems proposed in the literature that are more robust for homogeneous flow & in other more complicated context (examples)
5. In cases when individual pressure law differs in form (see below), new mixture pressure law should be devised first & construct model equations based on that

Barotropic & Non-Barotropic Flow

- Fluid component 1: Tait EOS

$$p(\rho) = (p_0 + \mathcal{B}) \left(\frac{\rho}{\rho_0}\right)^{\gamma} - \mathcal{B}$$

- Fluid component 2: Noble-Abel EOS

$$p(\rho, e) = \left(\frac{\gamma - 1}{1 - b\rho}\right) \rho e$$

- Mixture pressure law (Shyue, Shock Waves 2006)

$$p = \begin{cases} (p_0 + \mathcal{B}) \left(\dfrac{\rho}{\rho_0}\right)^{\gamma} - \mathcal{B} & \text{if} \quad \alpha = 1 \\ \left(\dfrac{\gamma - 1}{1 - b\rho}\right) (\rho e - \mathcal{B}) - \mathcal{B} & \text{if} \quad \alpha \neq 1 \end{cases}$$

Barotropic Two-Phase Flow

- Fluid component ι: Tait EOS

$$p(\rho) = (p_{0\iota} + \mathcal{B}_\iota)\left(\frac{\rho}{\rho_{0\iota}}\right)^{\gamma_\iota} - \mathcal{B}_\iota, \quad \iota = 1, 2$$

- Mixture pressure law (Shyue, JCP 2004)

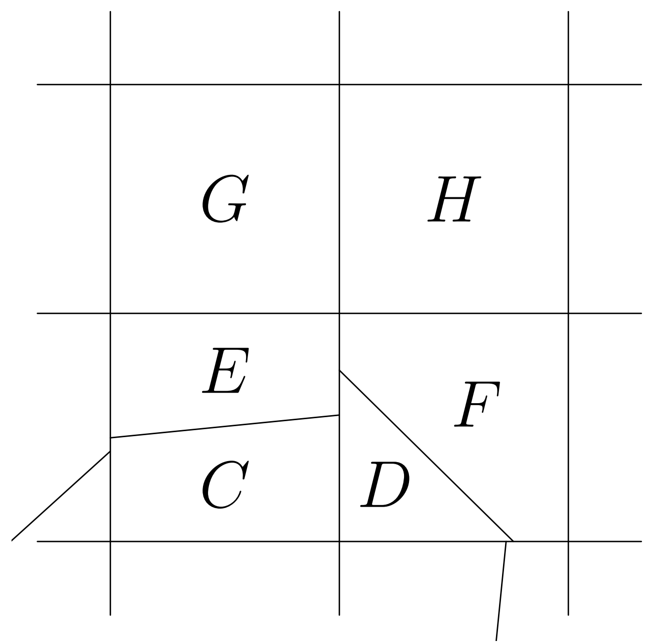
$$p = \begin{cases} (p_{0\iota} + \mathcal{B}_\iota)\left(\dfrac{\rho}{\rho_{0\iota}}\right)^{\gamma_\iota} - \mathcal{B}_\iota & \text{if} \quad \alpha = \alpha_\iota \ \text{(0 or 1)} \\ (\gamma - 1)\,\rho\left(e + \dfrac{\mathcal{B}}{\rho_0}\right) - \gamma\mathcal{B} & \text{if} \quad \alpha \in (0, 1) \end{cases}$$

Wave Propagation Method

Finite volume formulation of wave propagation method, Q_S^n gives approximate value of cell average of solution q over cell S at time t_n

$$Q_S^n \approx \frac{1}{\mathcal{M}(S)} \int_S q(X, t_n)\, dV$$

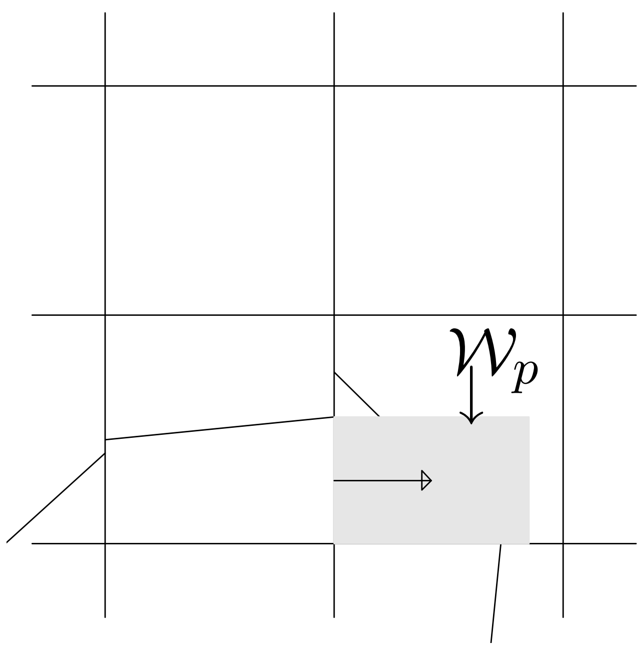
$\mathcal{M}(S)$: measure (area in 2D or volume in 3D) of cell S

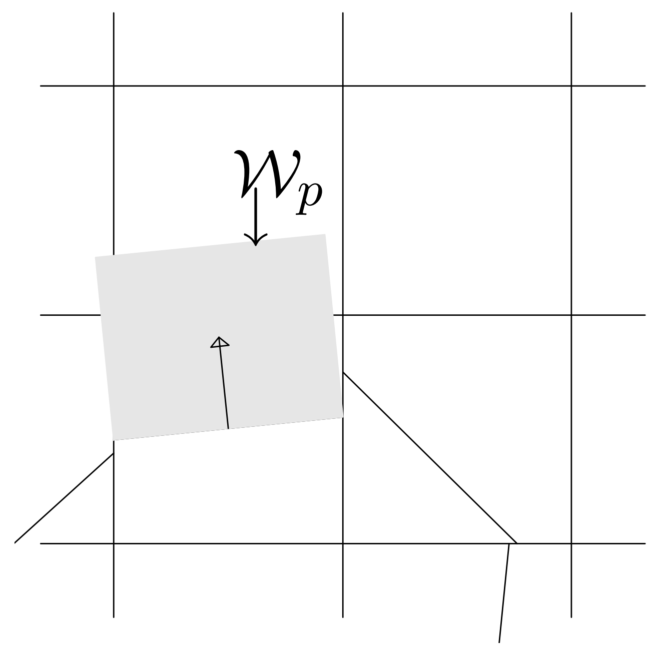


Wave Propagation Method (cont.)

- First order version: Piecewise constant wave update
  - Godunov-type method: Solve Riemann problem at each cell interface in normal direction & use resulting waves to update cell averages

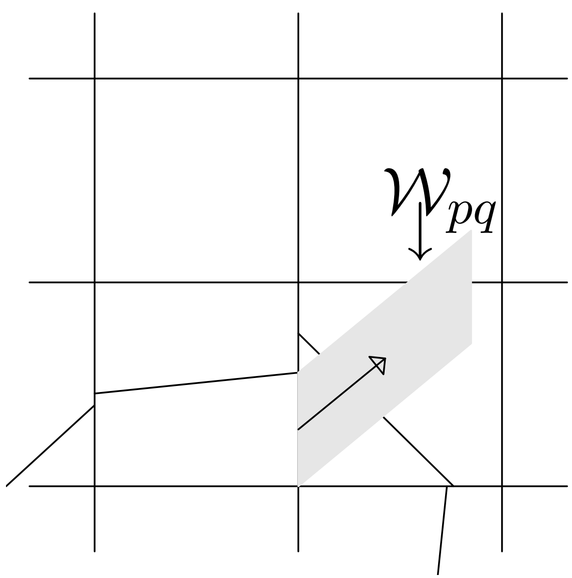
$$Q_S^{n+1} := Q_S^{n+1} - \frac{\mathcal{M}\left(\mathcal{W}_p \cap S\right)}{\mathcal{M}(S)} R_p, \qquad R_p \text{ being jump from RP}$$

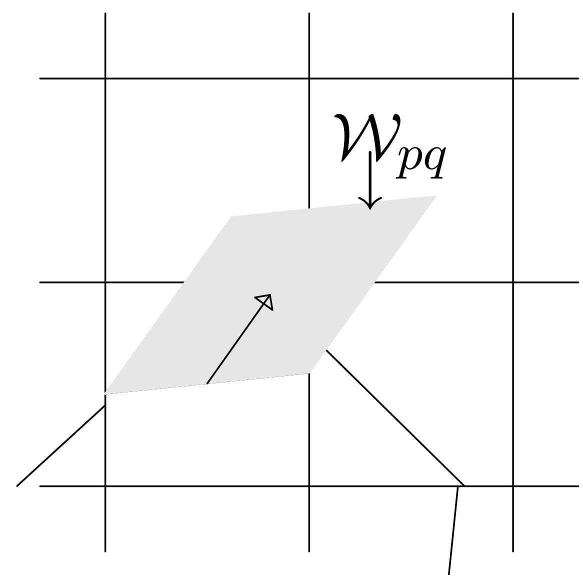




Wave Propagation Method (cont.)

- First order version: Transverse-wave included
  - Use transverse portion of equation, solve Riemann problem in transverse direction, & use resulting waves to update cell averages as usual
  - Stability of method is typically improved, while conservation of method is maintained

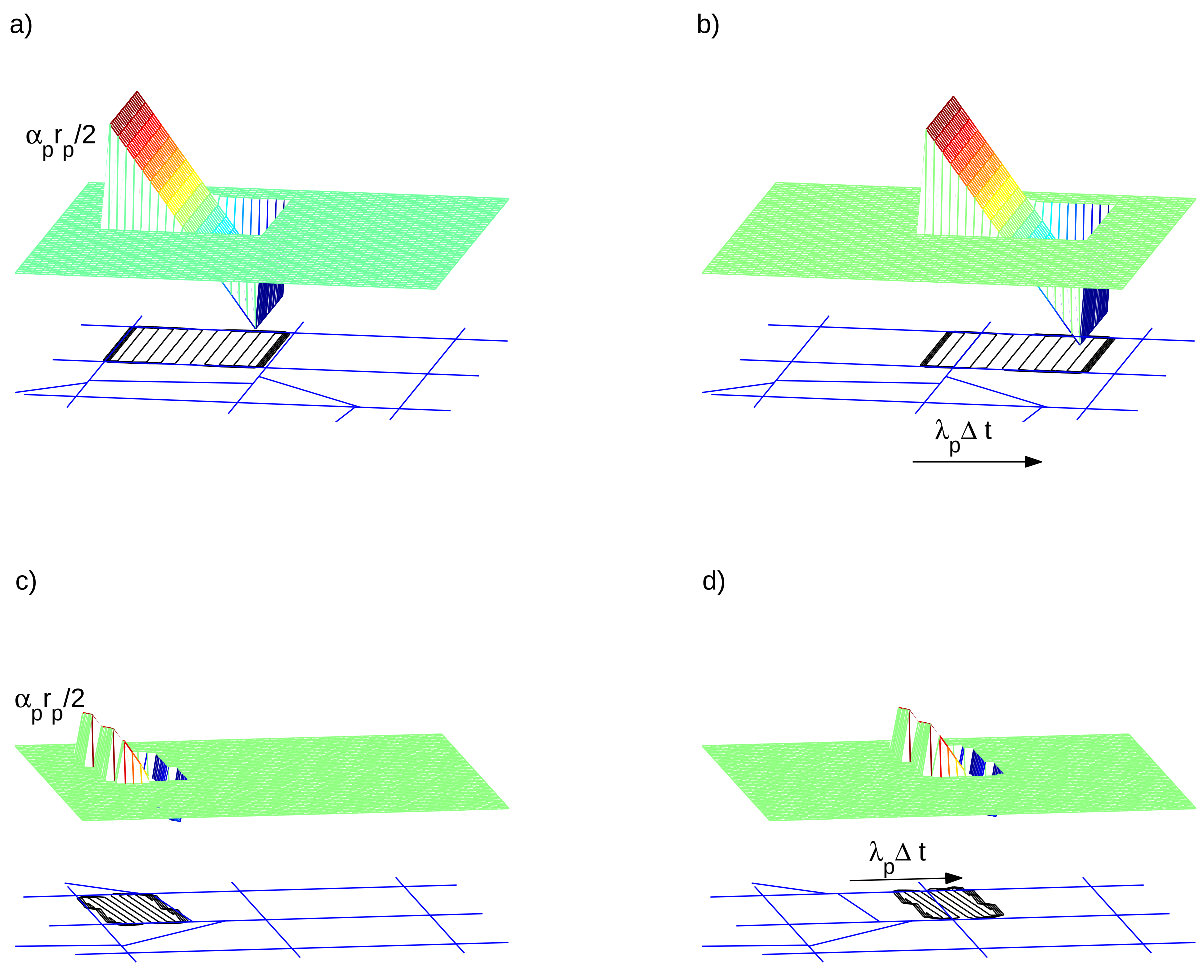




Wave Propagation Method (cont.)

- High resolution version: Piecewise linear wave update

wave before propagation after propagation



Volume Tracking Algorithm

1. Volume moving procedure
   (a) *Volume fraction update*
   Take a time step on current grid to update cell averages of volume fractions at next time step
   (b) *Interface reconstruction*
   Find new interface location based on volume fractions obtained in (a) using an interface reconstruction scheme. Some cells will be subdivided & values in each subcell must be initialized.
2. Physical solution update
   Take same time interval as in (a), but use a method to update cell averages of multicomponent model on new grid created in (b)

Interface Reconstruction Scheme

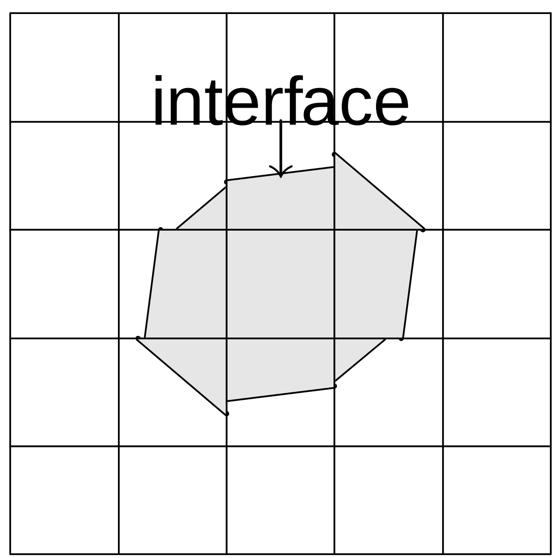
Given volume fractions on current grid, piecewise linear interface reconstruction (PLIC) method does:

1. Compute interface normal
   - Gradient method of Parker & Youngs
   - Least squares method of Puckett
2. Determine interface location by iterative bisection

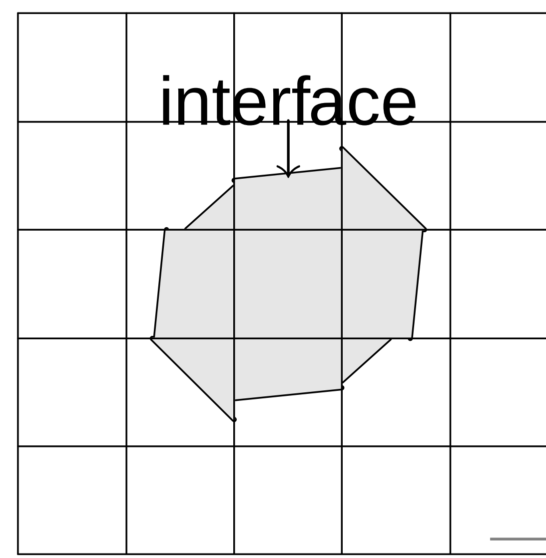
Data set

0	0	0	0	0
0	0.09	0.51	0.29	0
0	0.68	1	0.68	0
0	0.29	0.51	0.09	0
0	0	0	0	0

Parker & Youngs



Puckett



Volume Moving Procedure

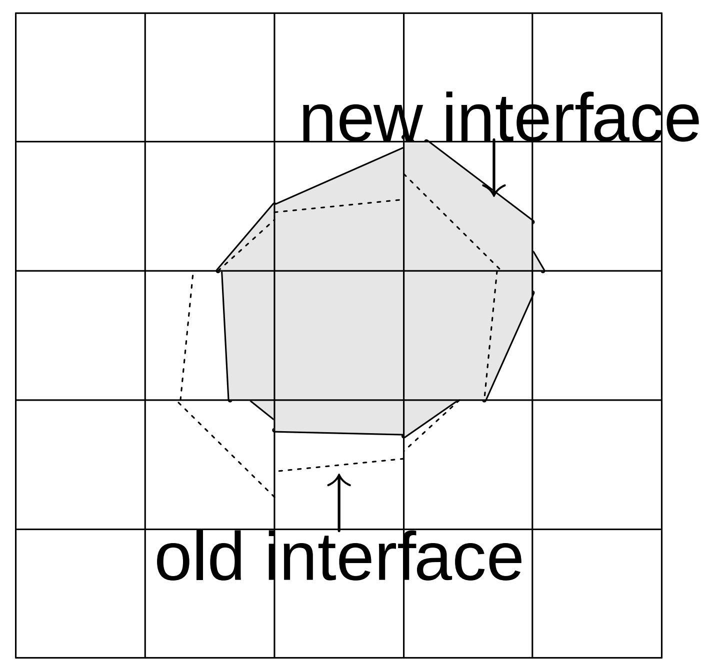
(a) Volume fractions given in previous slide are updated with uniform $(u, v) = (1, 1)$ over $\Delta t = 0.06$

(b) New interface location is reconstructed

(a)

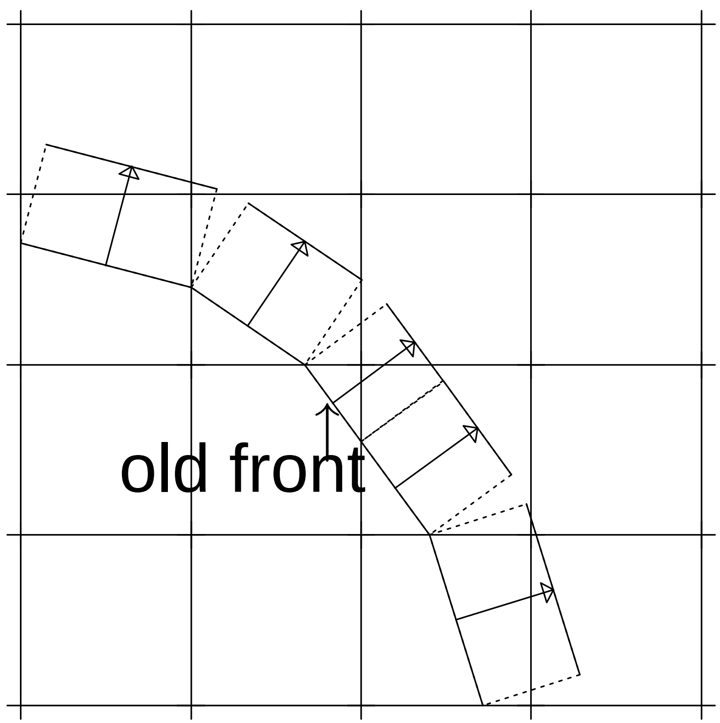
0	0	0	$1(-3)$	0
0	0.11	0.72	0.74	$5(-3)$
0	0.38	1	0.85	0
0	0.01	0.25	0.06	0
0	0	0	0	0

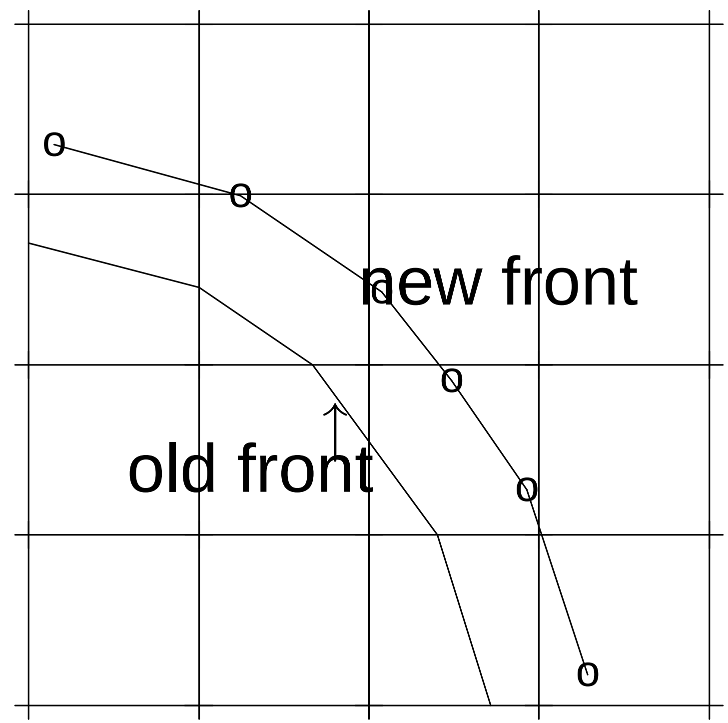
(b)



Surface Moving Procedure

Solve Riemann problem at tracked interfaces & use resulting wave speed of the tracked wave family over Δt to find new location of interface at the next time step





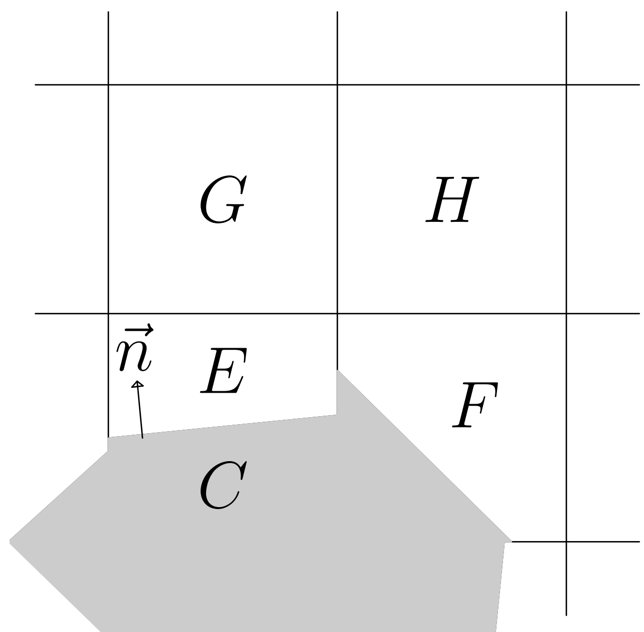
Boundary Conditions

For tracked segments representing rigid (solid wall) boundary (stationary or moving), reflection principle is used to assign states for fictitious subcells in each time step:

$$z_C := z_E \qquad (z = \rho, p, \alpha)$$

$$\vec{u}_C := \vec{u}_E - 2(\vec{u}_E \cdot \vec{n})\vec{n} + 2(\vec{u}_0 \cdot \vec{n})$$

$\vec{u}_0$: moving boundary velocity



Interface Conditions

For tracked segments representing material interfaces, pressure equilibrium as well as velocity continuity conditions across interfaces are fulfilled by

1. Devise of the wave-propagation method
2. Choice of Riemann solver used in the method

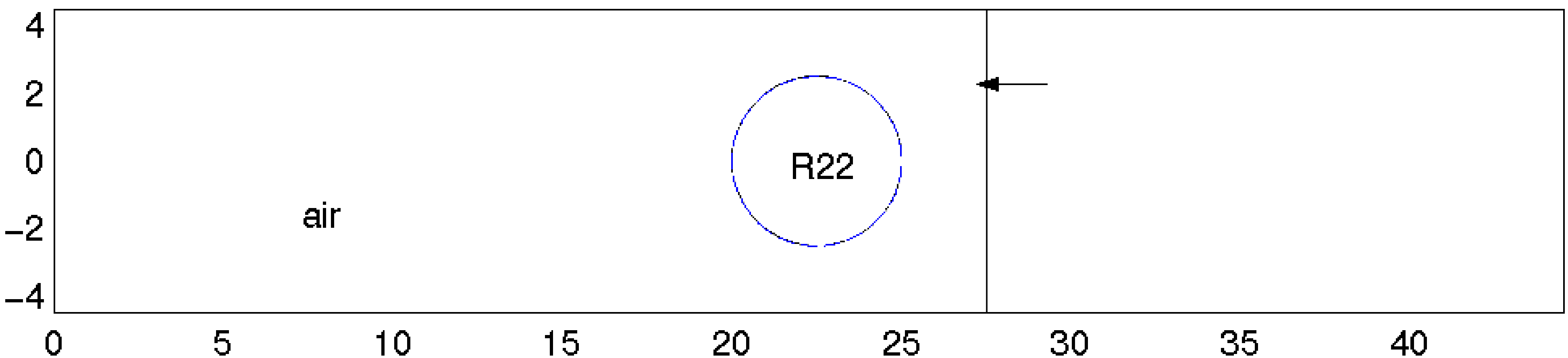
Stability Issues

- Choose time step Δt based on uniform grid mesh size Δx, Δy as

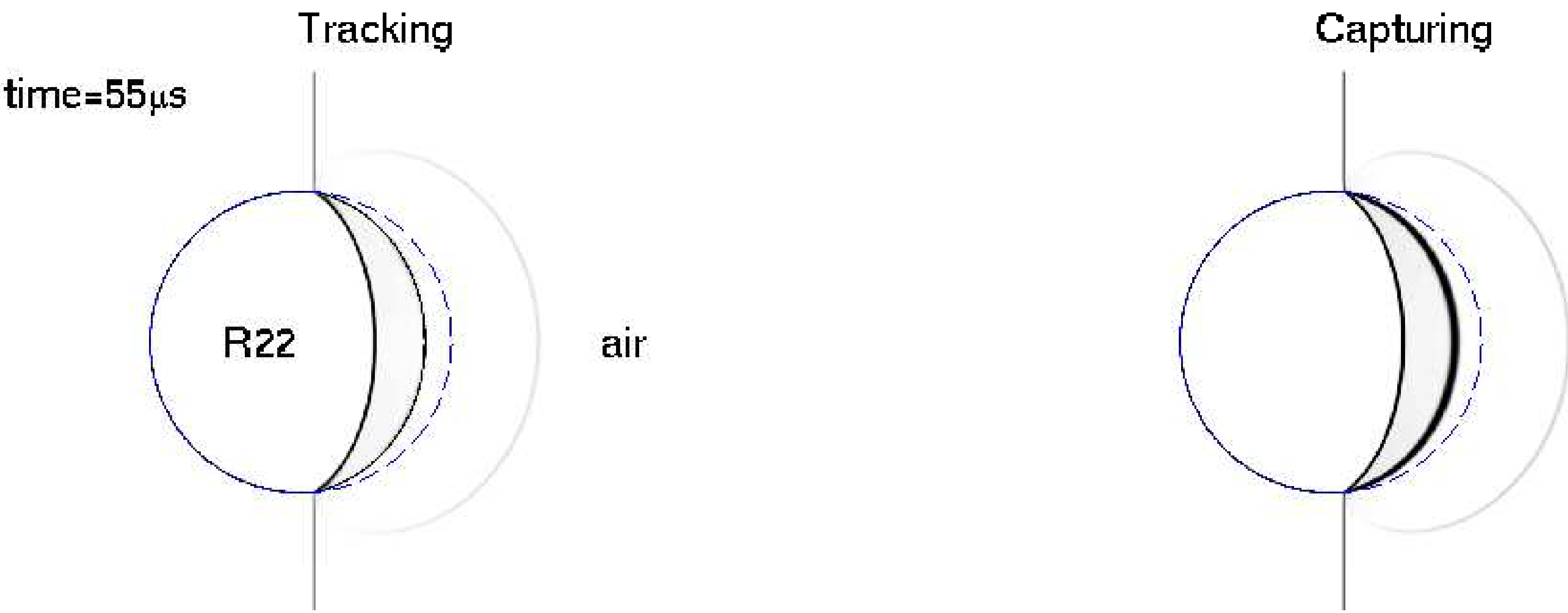
$$\frac{\Delta t \ \max_{p,q}(\lambda_p, \mu_q)}{\min(\Delta x, \Delta y)} \le 1,$$

  - λ_p, μ_q: speed of p-wave, q-wave from Riemann problem solution in normal-, transverse-directions
- Use large time step method of LeVeque (*i.e.*, wave interactions are assumed to behave in linear manner) to maintain stability of method even in the presence of small Cartesian cut cells
- Apply smoothing operator (such as, h-box approach of Berger *et al.*) locally for cell averages in irregular cells

Shock-Bubble Interaction Problem



Shock-Bubble Interaction Problem



Shock-Bubble Interaction Problem

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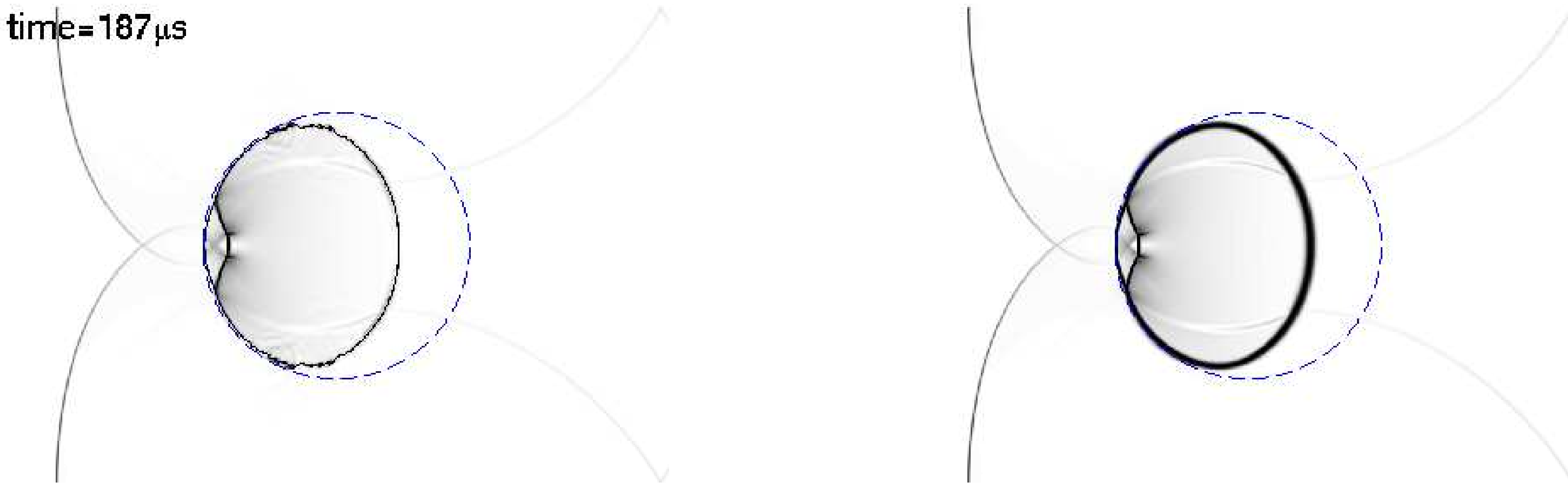
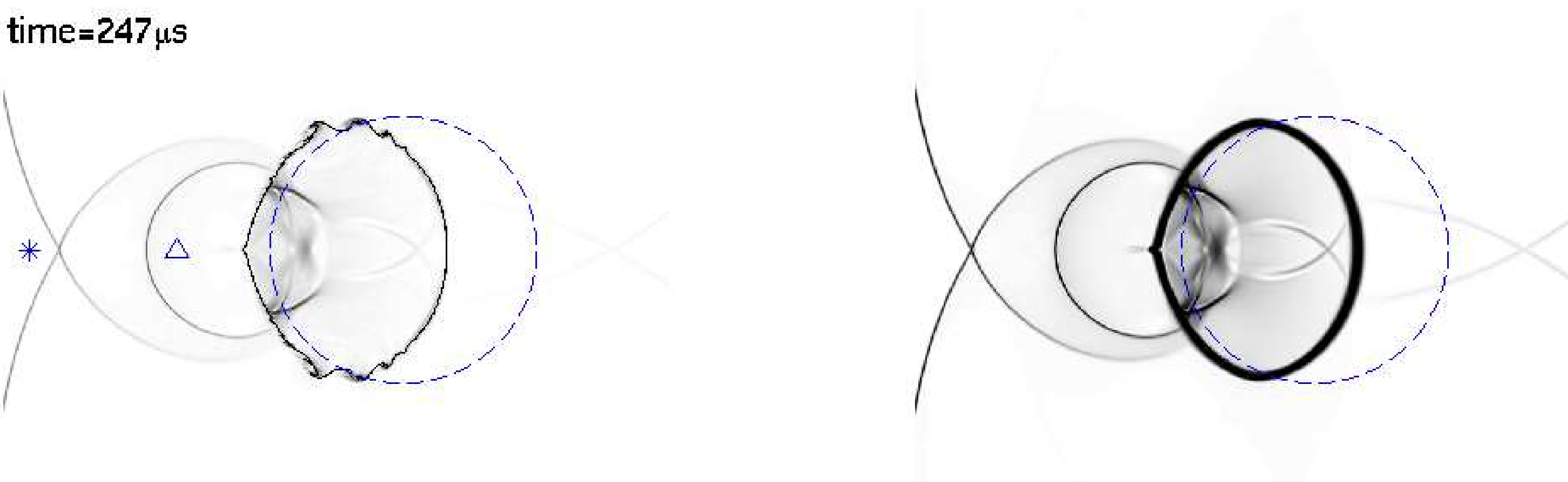


Shock-Bubble Interaction Problem



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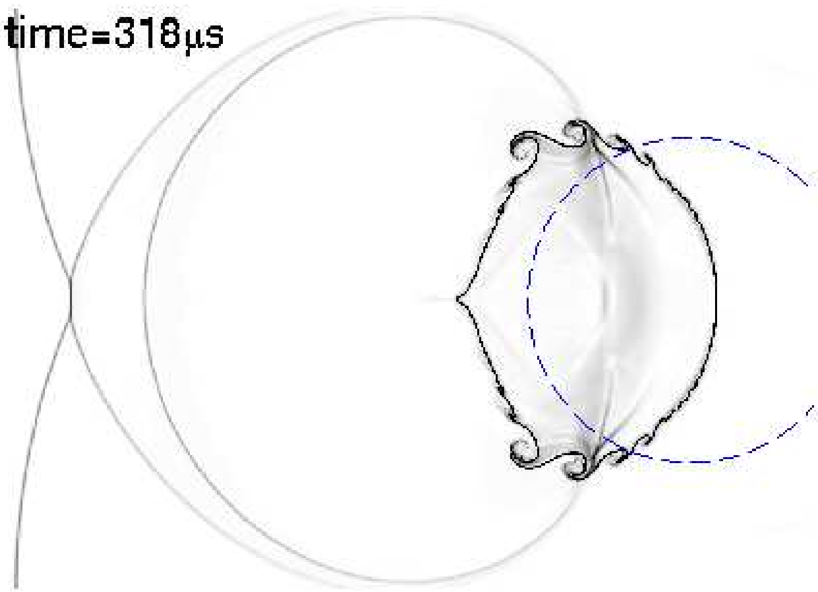
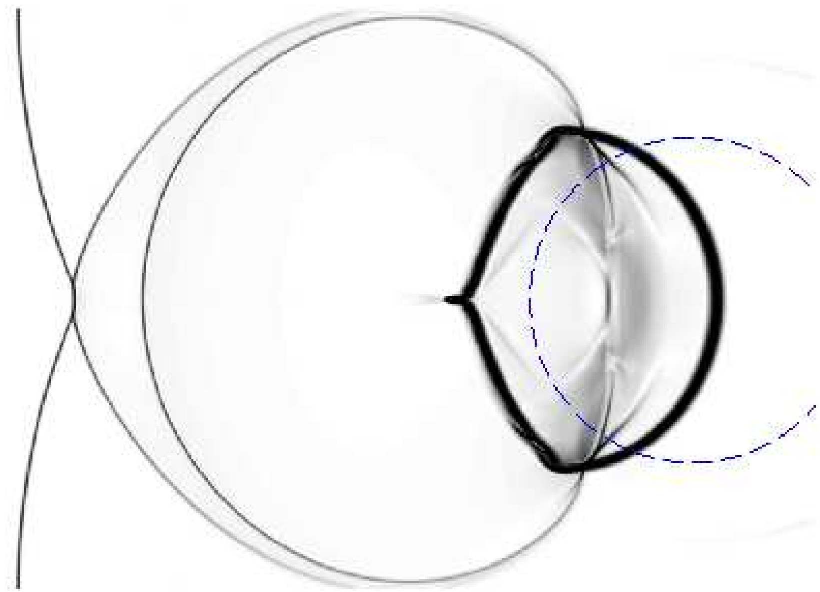
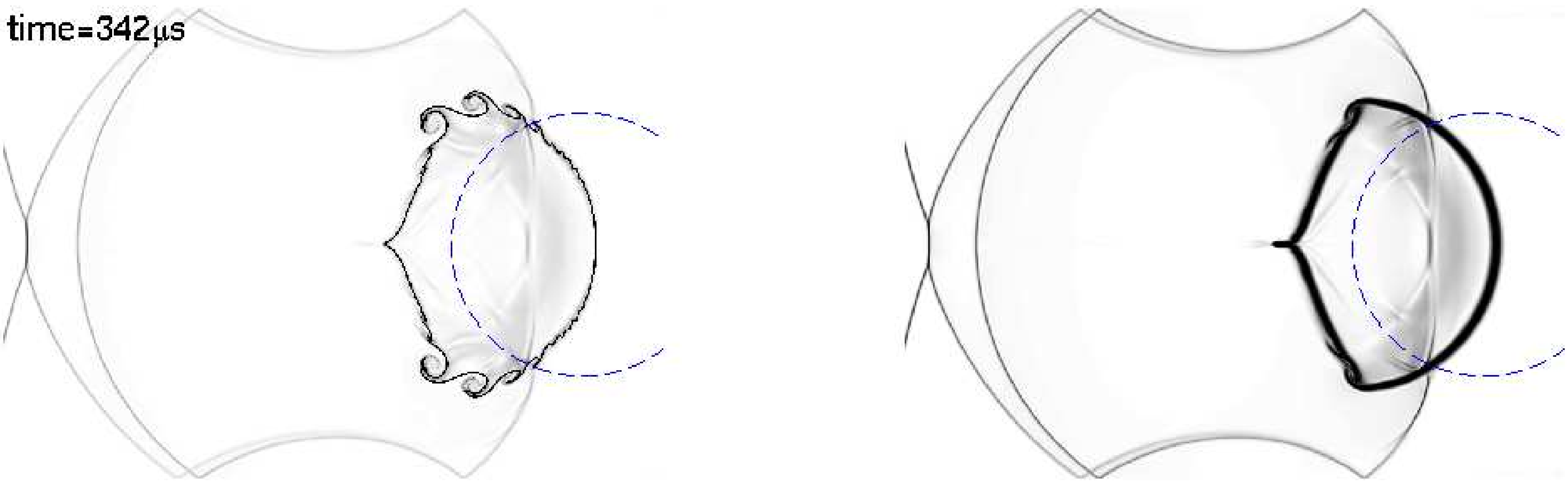
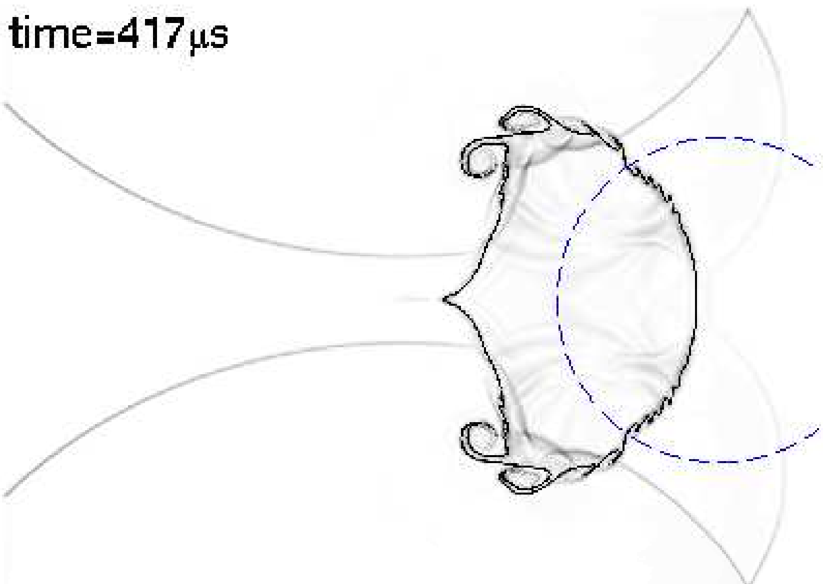


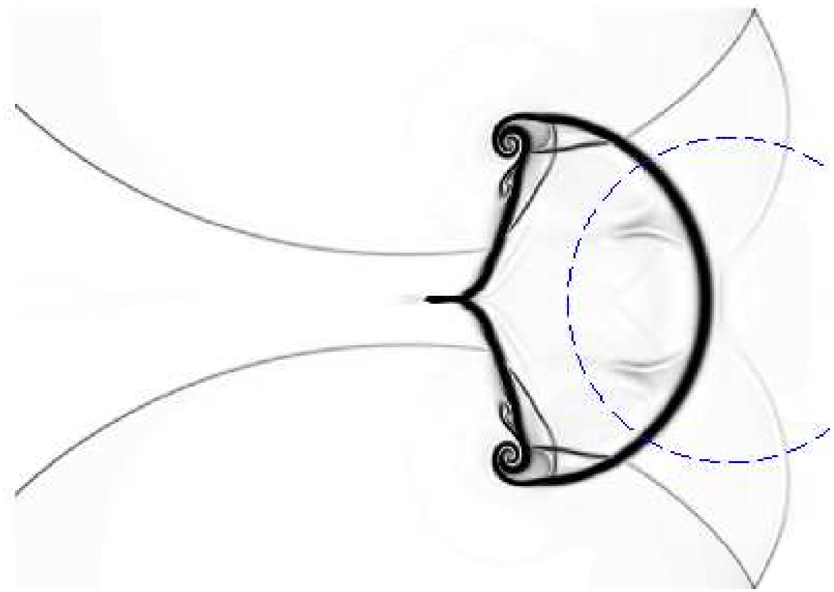


Shock-Bubble Interaction Problem



Shock-Bubble Interaction Problem





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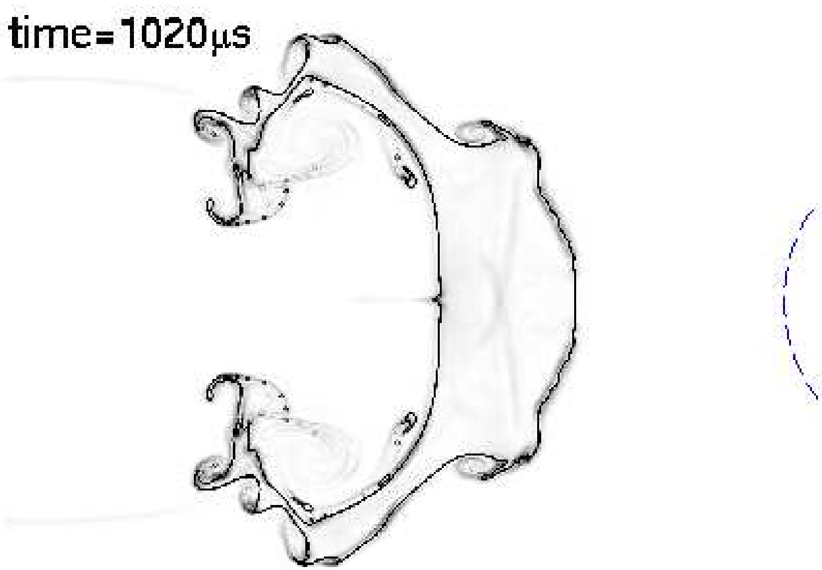
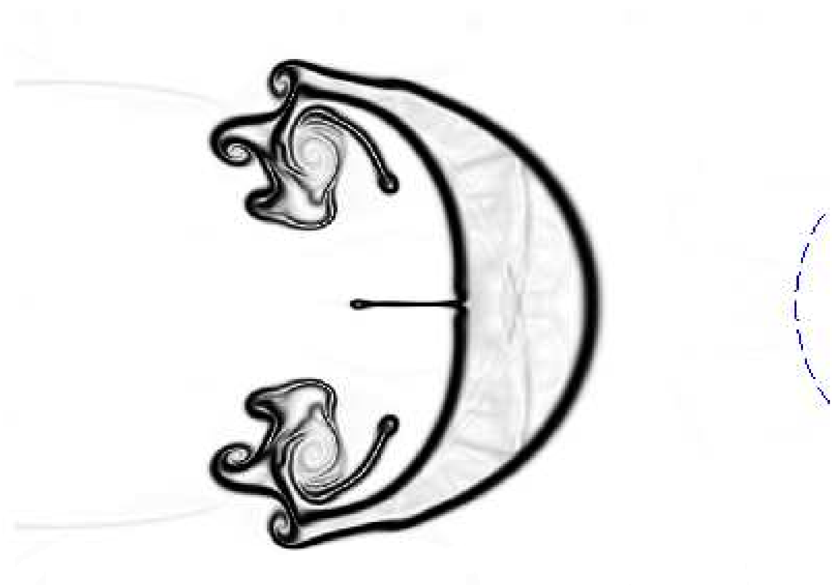
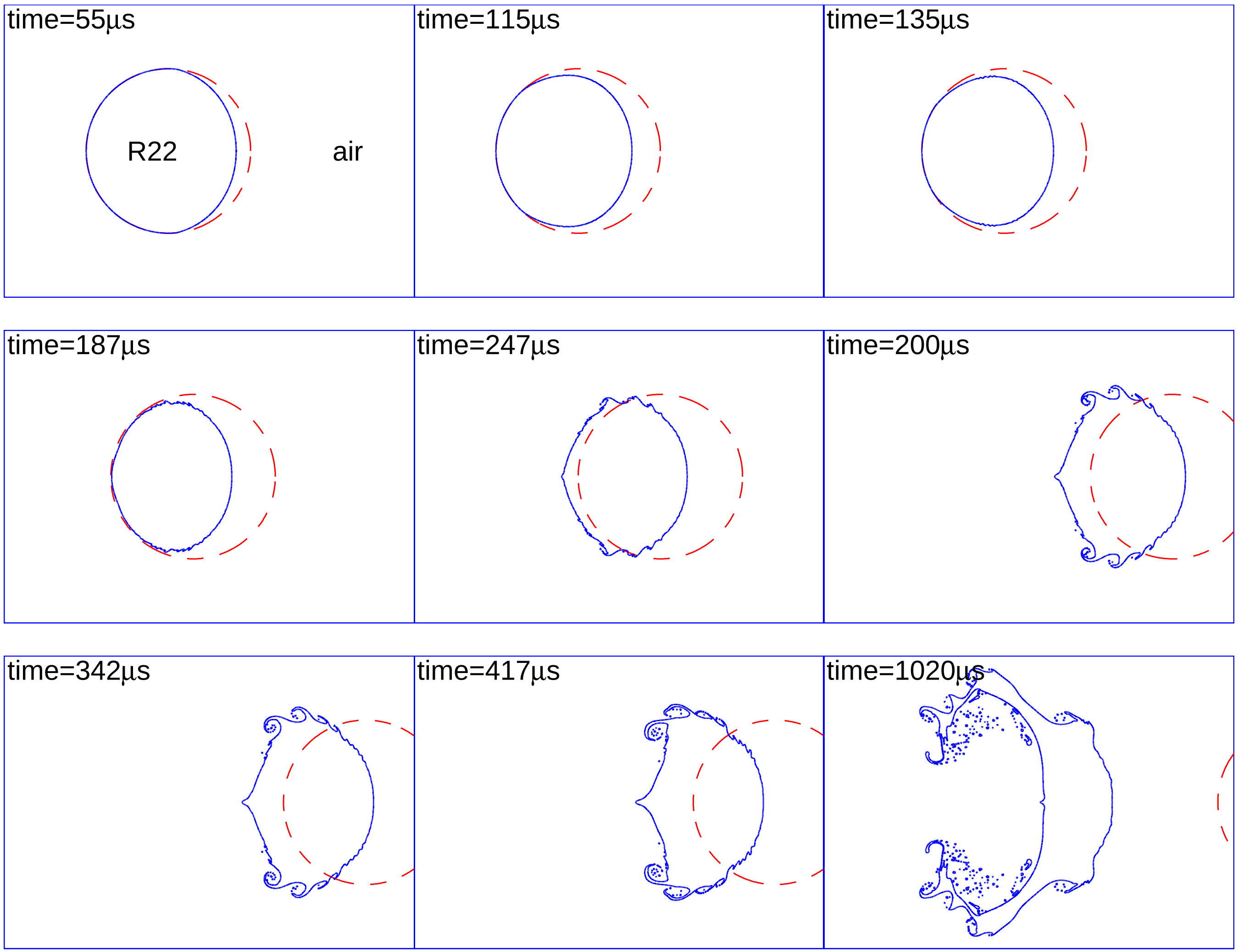




Shock-Bubble Interaction (cont.)

- Approximate locations of interfaces



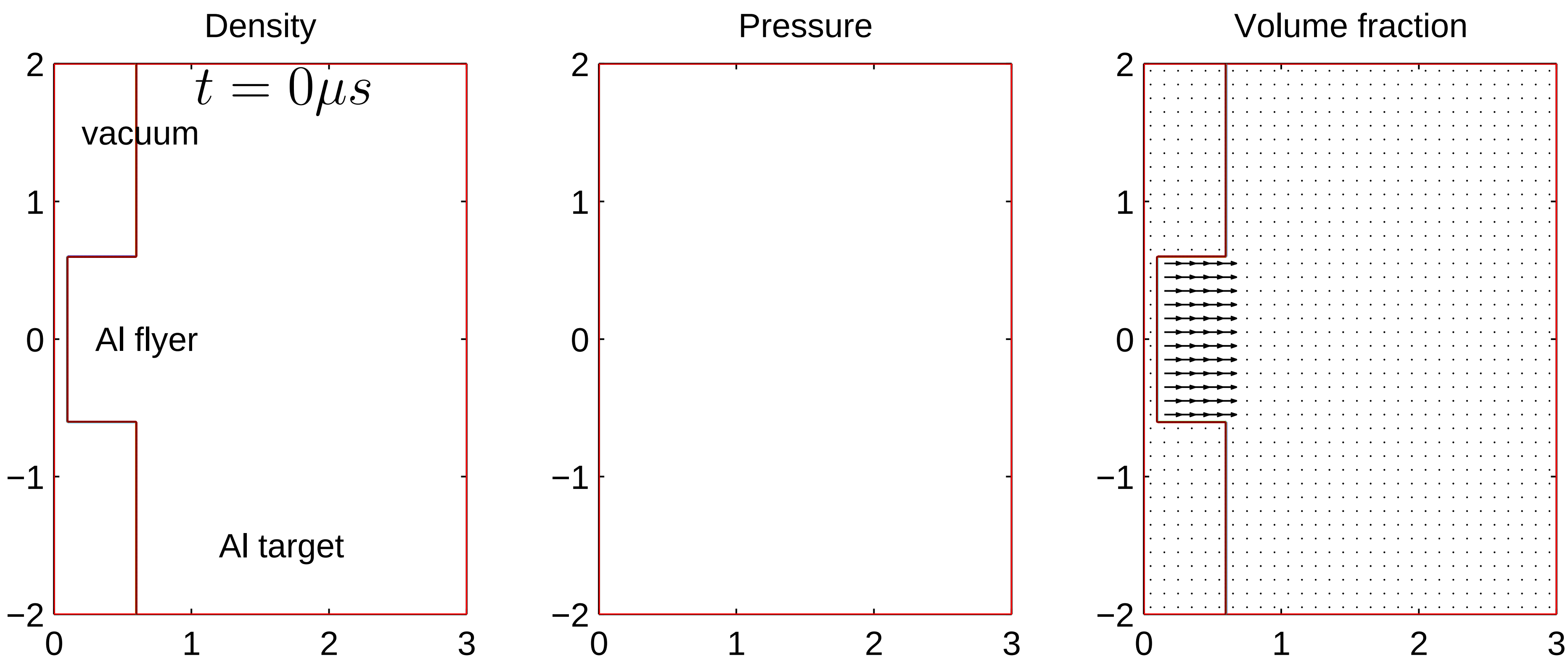
Shock-Bubble Interaction (cont.)

- Quantitative assessment of prominent flow velocities:

Velocity (m/s)	V_s	V_R	V_T	V_{ui}	V_{uf}	V_{di}	V_{df}
Haas & Sturtevant	415	240	540	73	90	78	78
Quirk & Karni	420	254	560	74	90	116	82
Our result (tracking)	411	243	538	64	87	82	60
Our result (capturing)	411	244	534	65	86	98	76

- V_s (V_R, V_T) Incident (refracted, transmitted) shock speed $t \in [0, 250]\mu$s ($t \in [0, 202]\mu$s, $t \in [202, 250]\mu$s)
- V_{ui} (V_{uf}) Initial (final) upstream bubble wall speed $t \in [0, 400]\mu$s ($t \in [400, 1000]\mu$s)
- V_{di} (V_{df}) Initial (final) downstream bubble wall speed $t \in [200, 400]\mu$s ($t \in [400, 1000]\mu$s)

Aluminum-Plate Impact Problem



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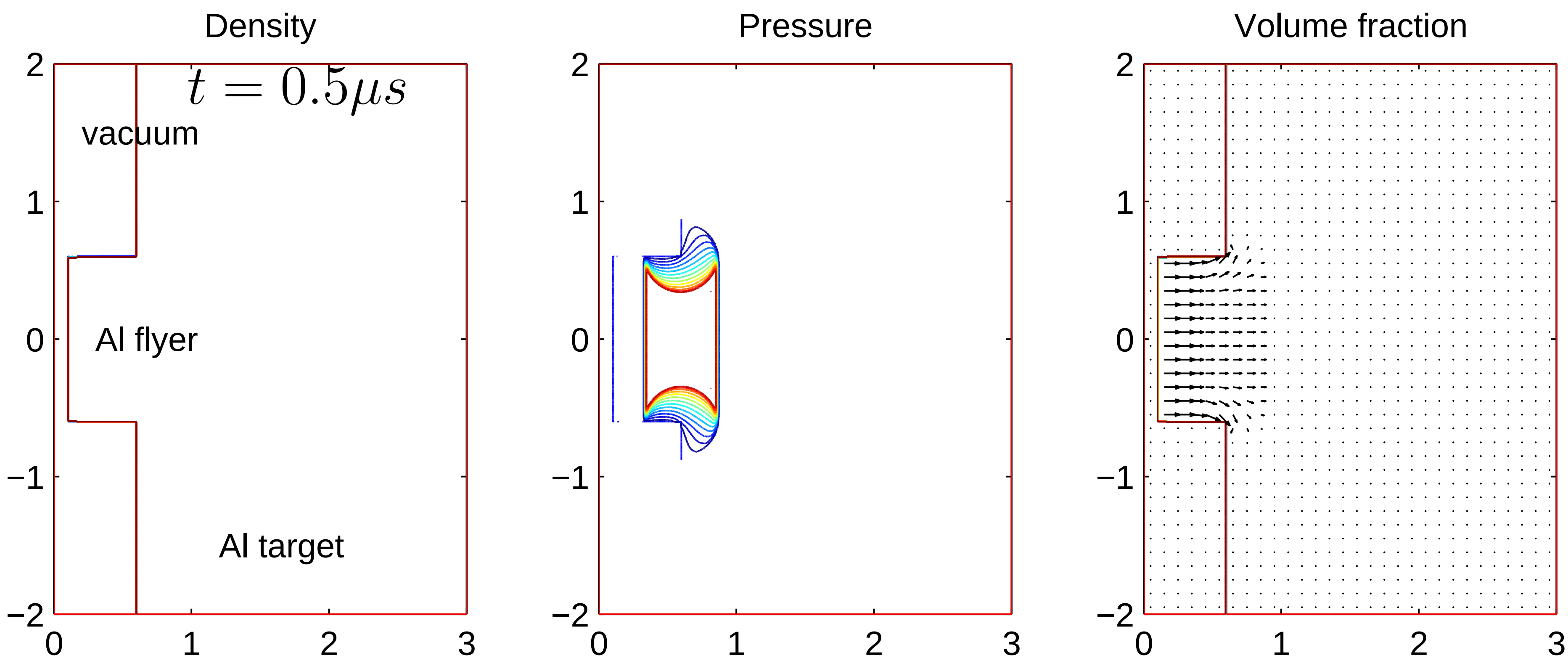

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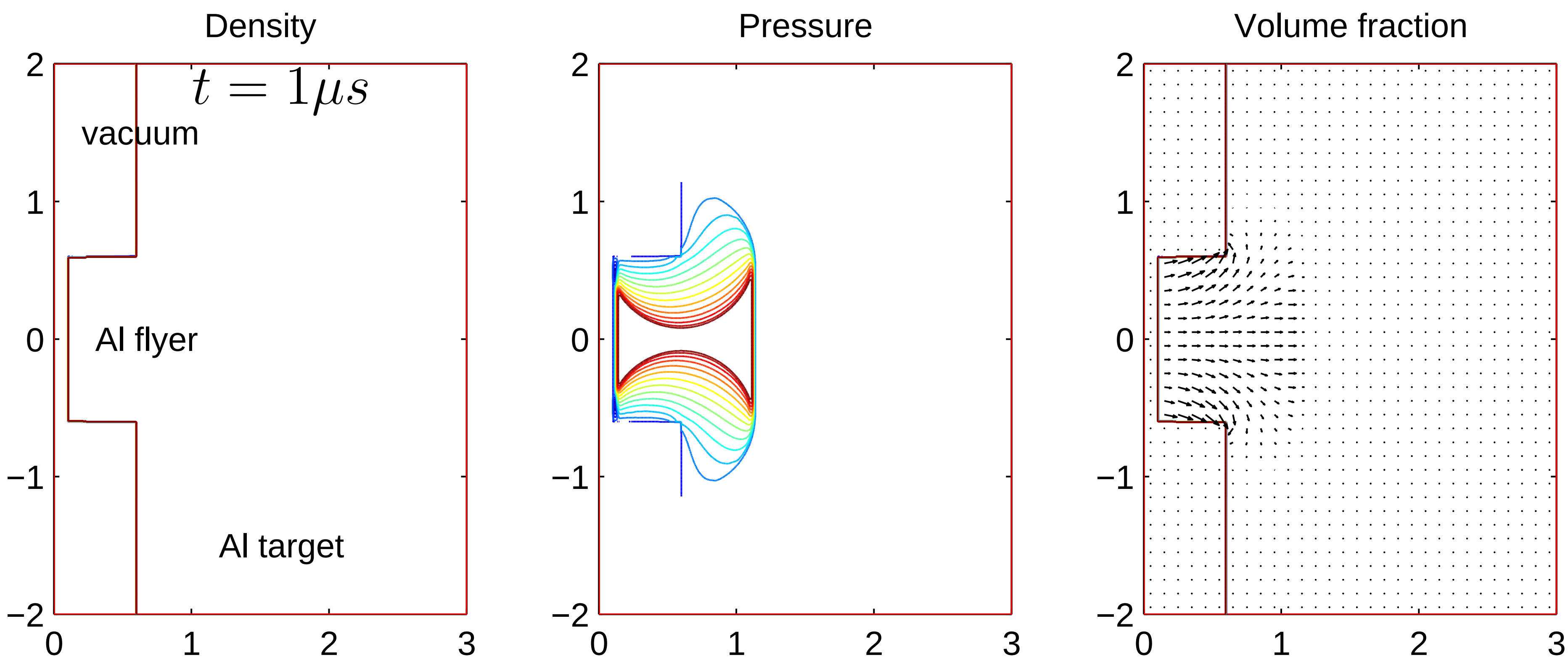

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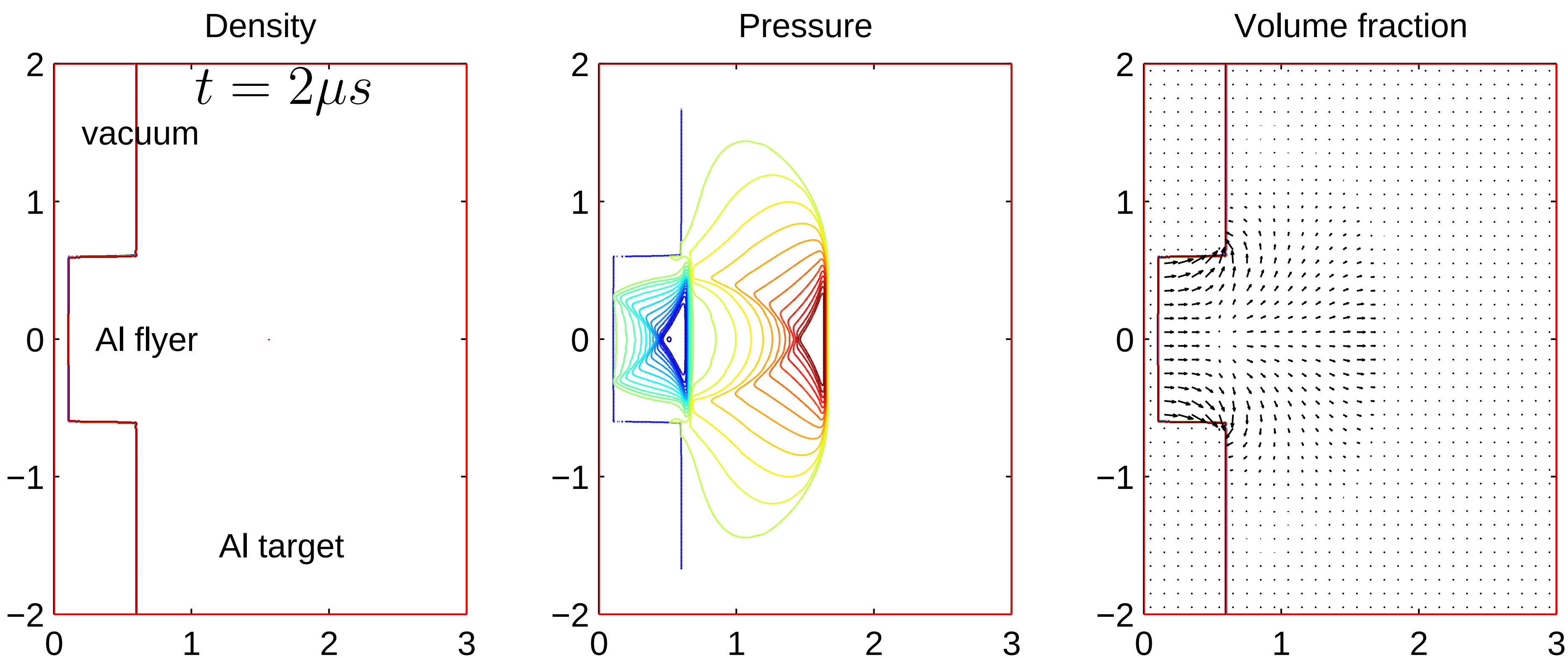

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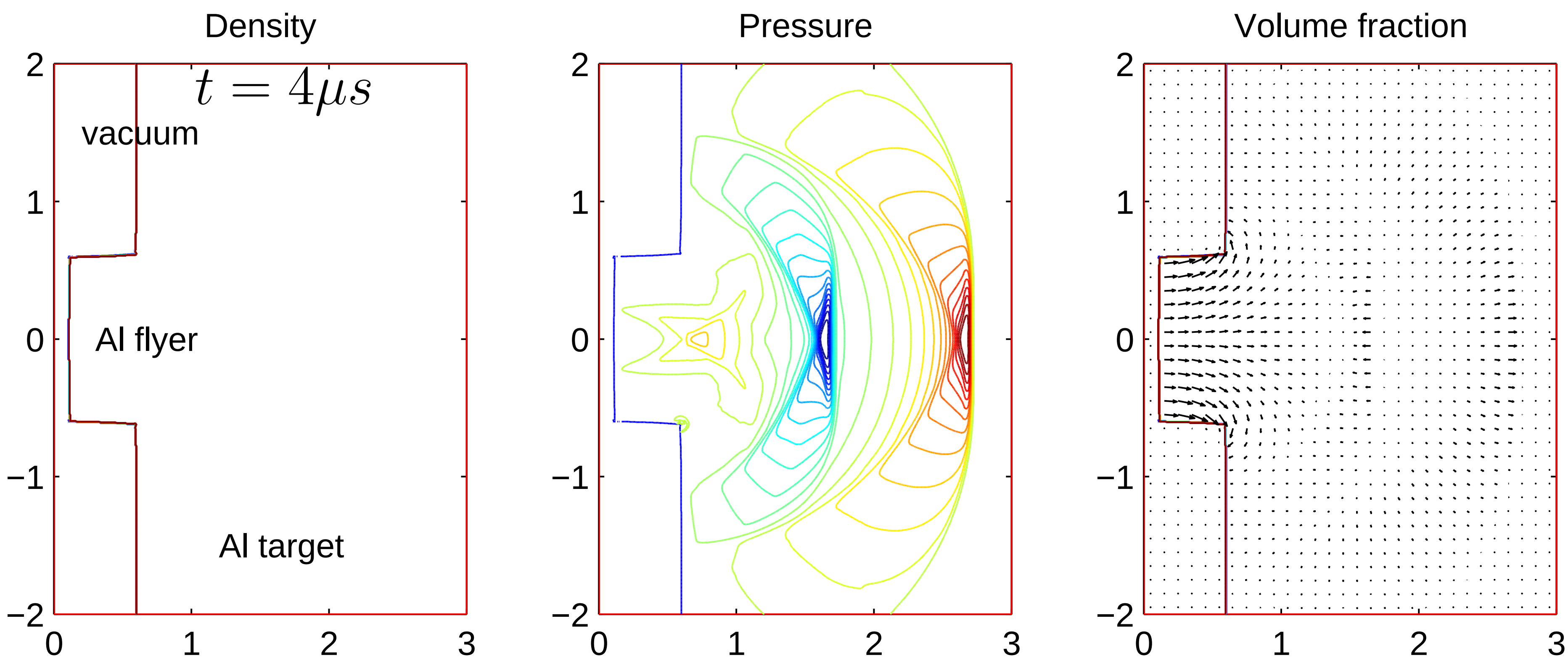
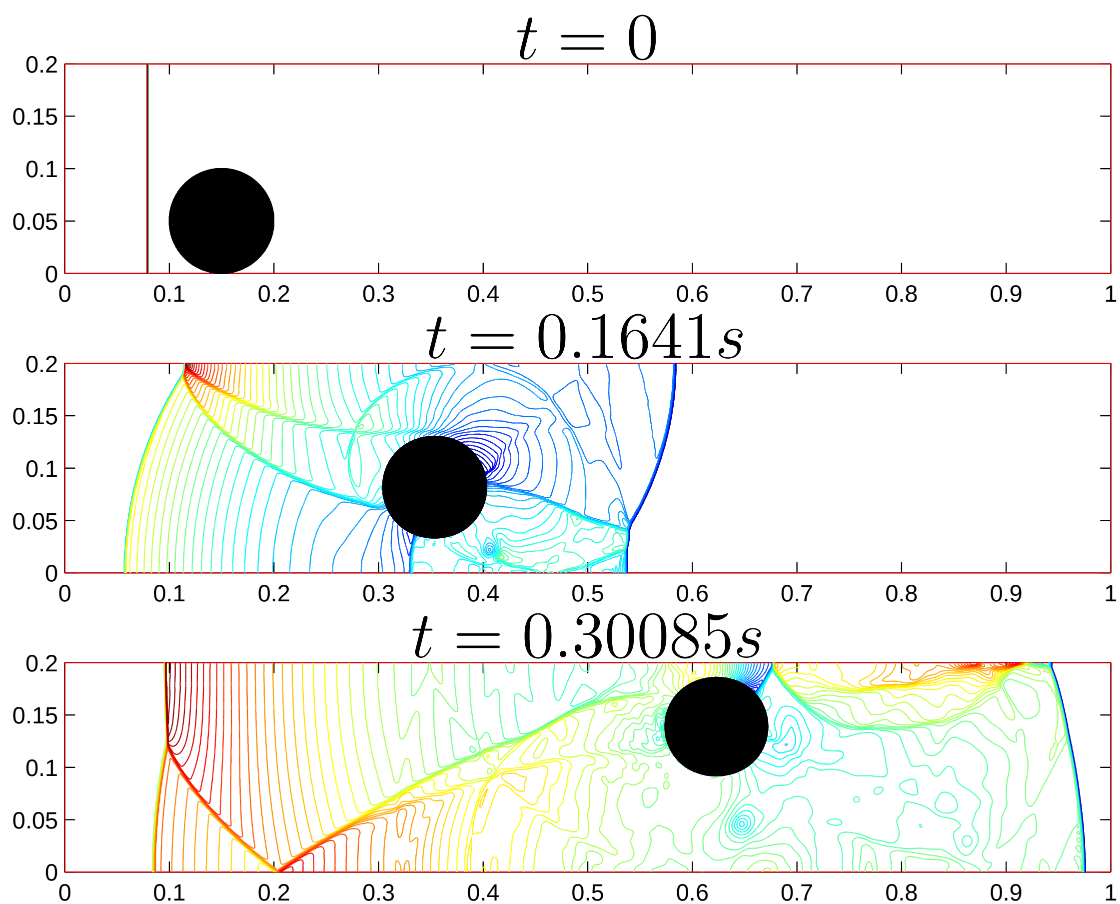


Cylinder lift-off Problem

- Moving speed of cylinder is governed by Newton's law
- Pressure contours are shown with a 1000×200 grid



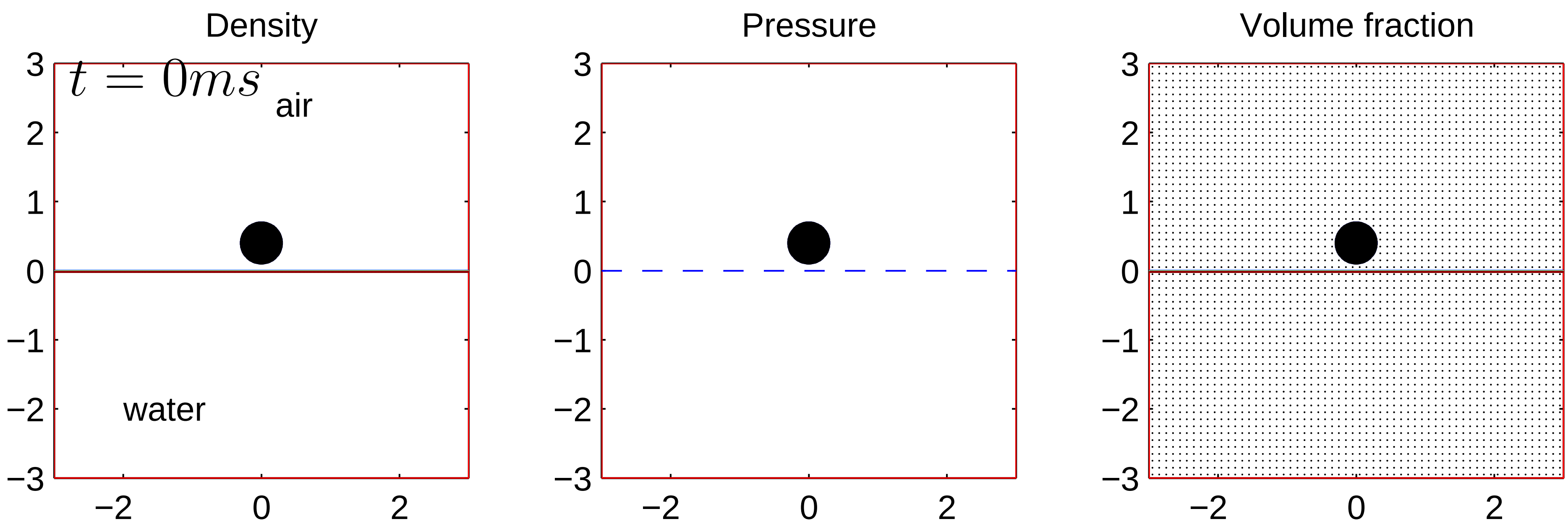
Cylinder lift-off Problem

- A convergence study of center of cylinder & relative mass loss for at final stopping time $t = 0.30085s$

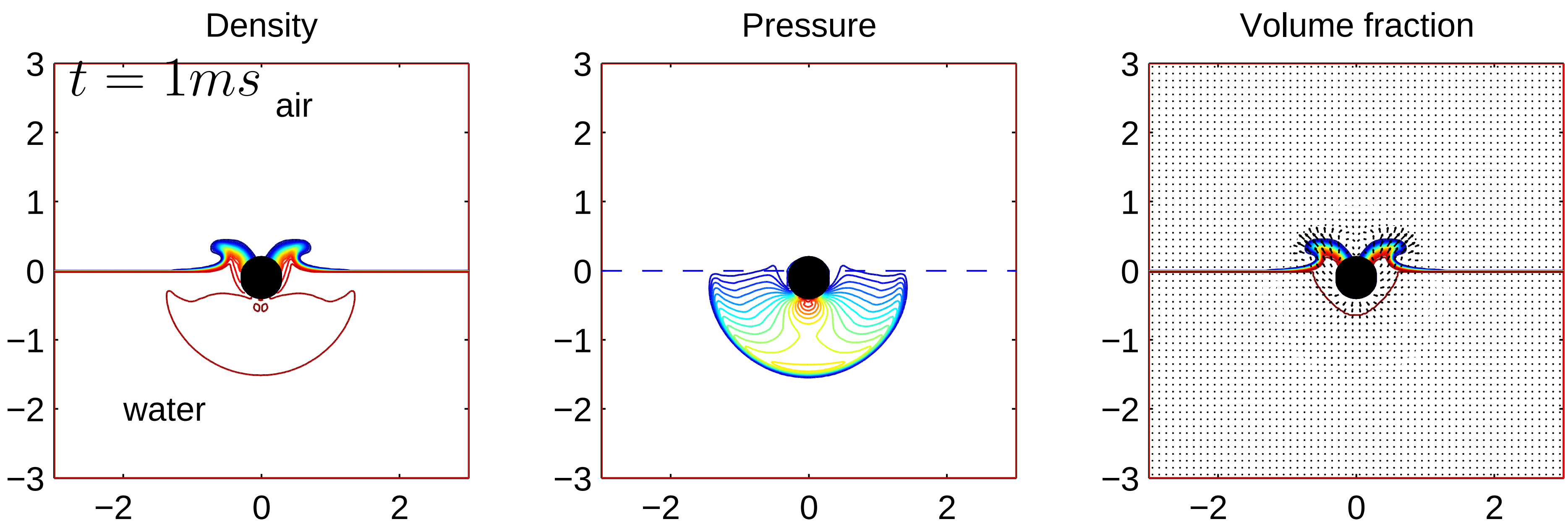
Mesh size	Center of cylinder	Relative mass loss
250×50	$(0.618181, 0.134456)$	-0.257528
500×100	$(0.620266, 0.136807)$	-0.131474
1000×200	$(0.623075, 0.138929)$	-0.066984

- Results are comparable with numerical appeared in literature

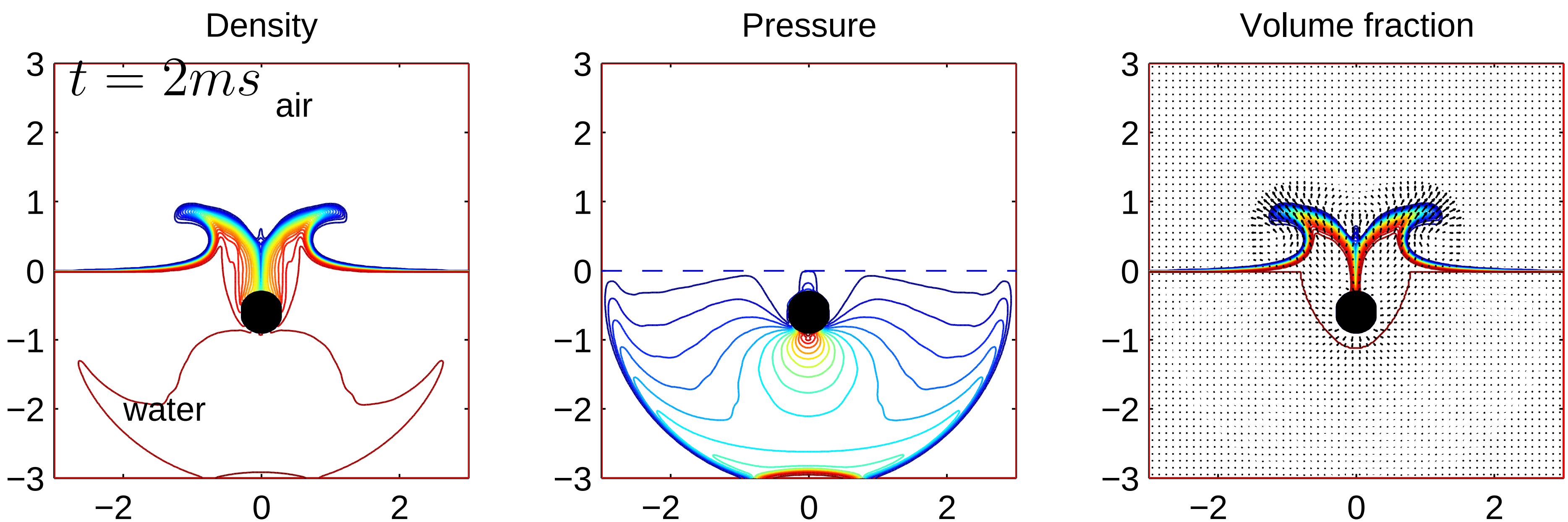
Falling Rigid Object in Water Tank



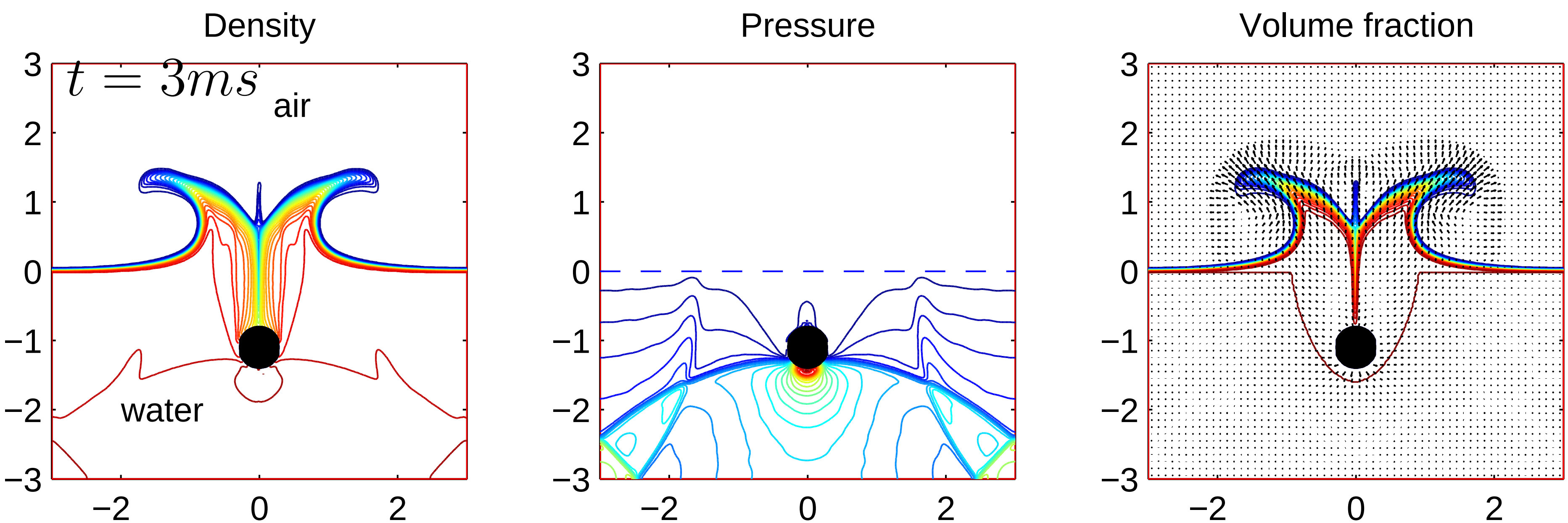
Falling Rigid Object in Water Tank



Falling Rigid Object in Water Tank



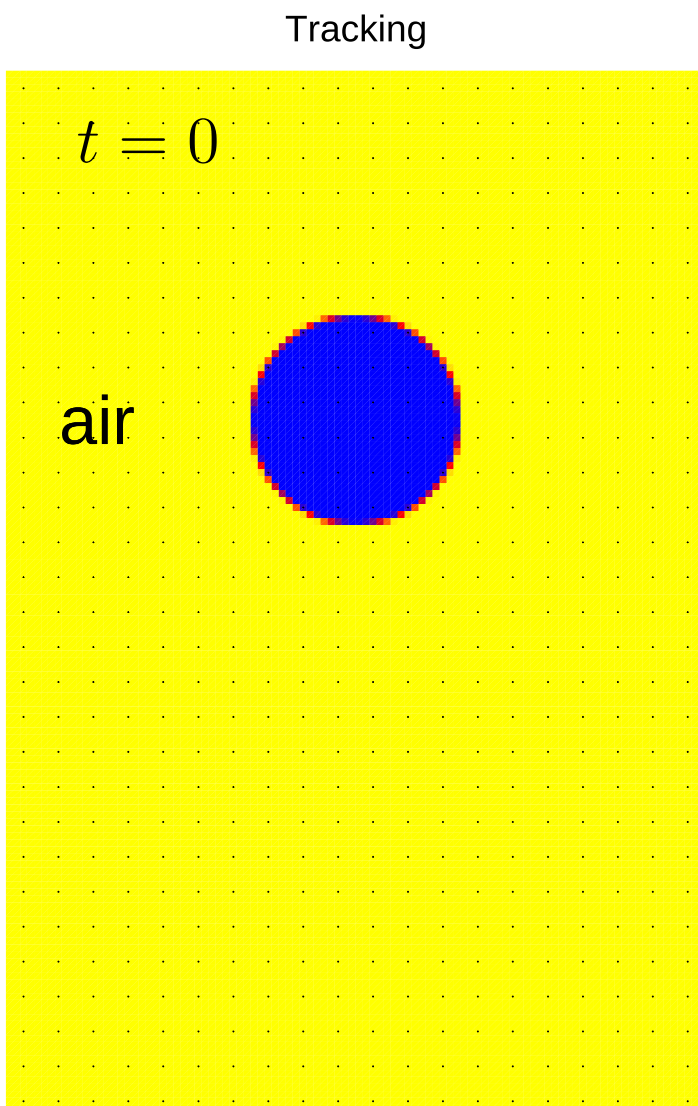
Falling Rigid Object in Water Tank

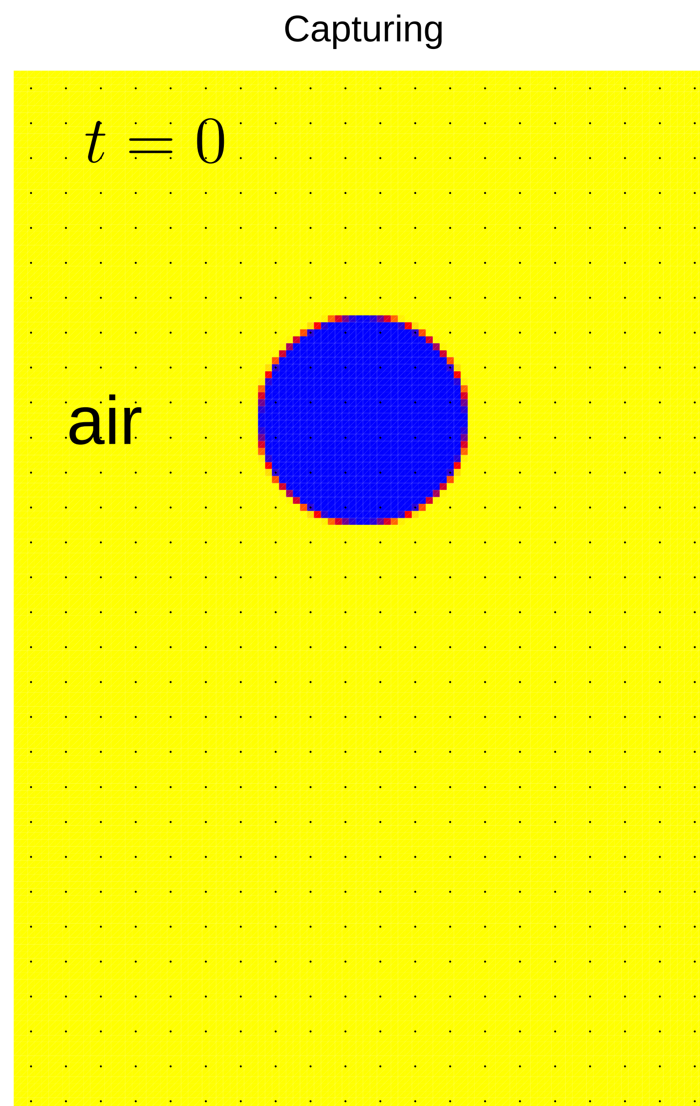


Future Work

- 3D volume tracking method
- General curvilinear grid system
  - Body-fitted grid for complicated geometries
- Low Mach number flow
  - Remove sound-speed stiffness by preconditioning techniques or pressure-based method
- Include more physics towards real applications
  - Diffusion, phase transition, or elastic-plastic effect
- Hybrid surface-volume tracking algorithm for balance laws with interfaces & boundaries

Liquid Drop Problem (Revisit)





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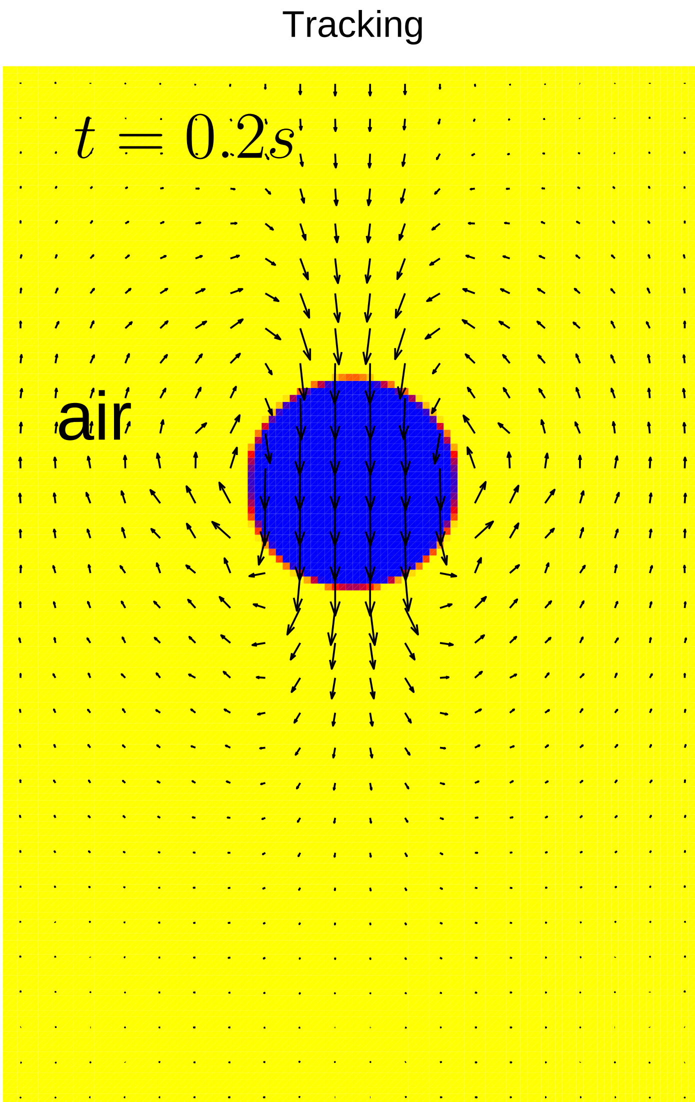

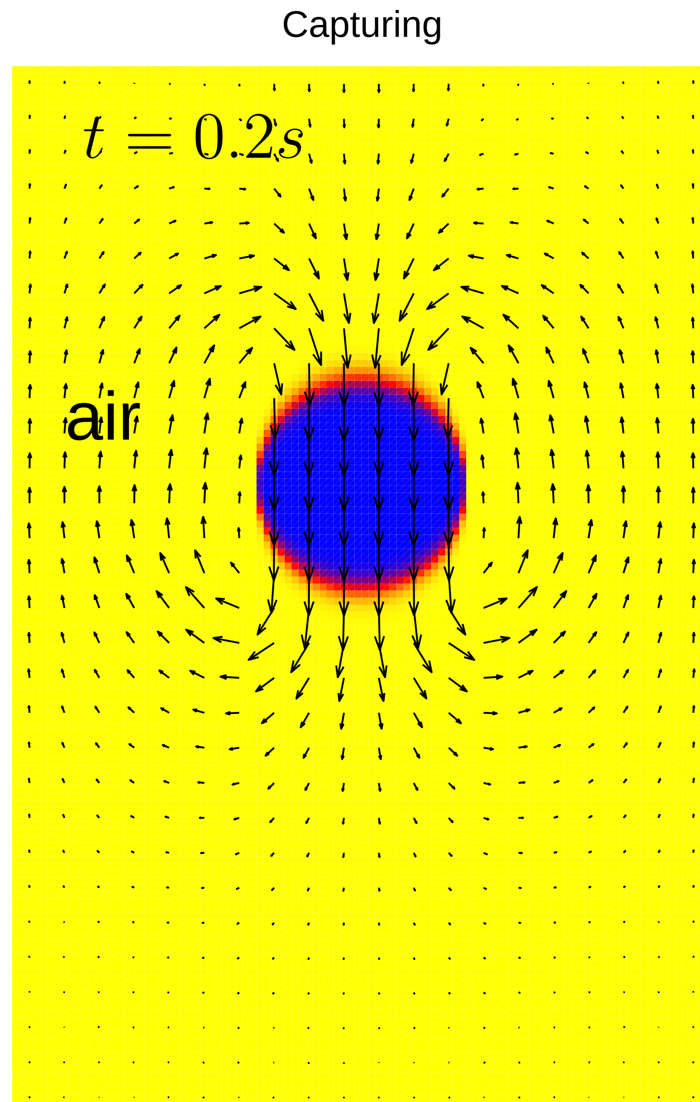

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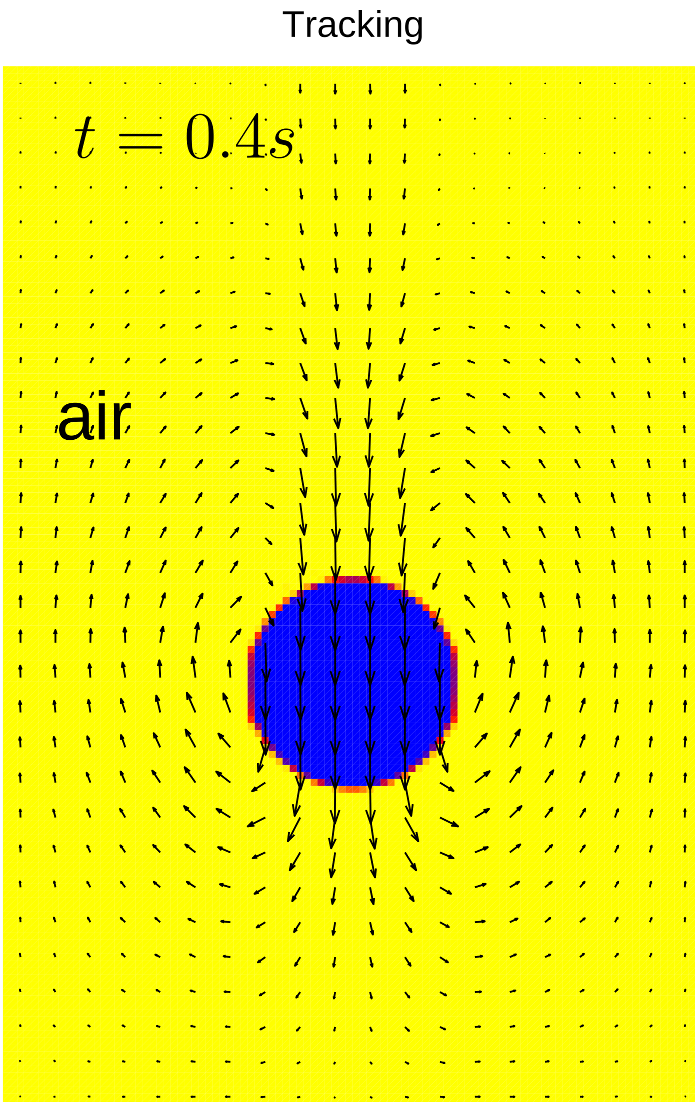

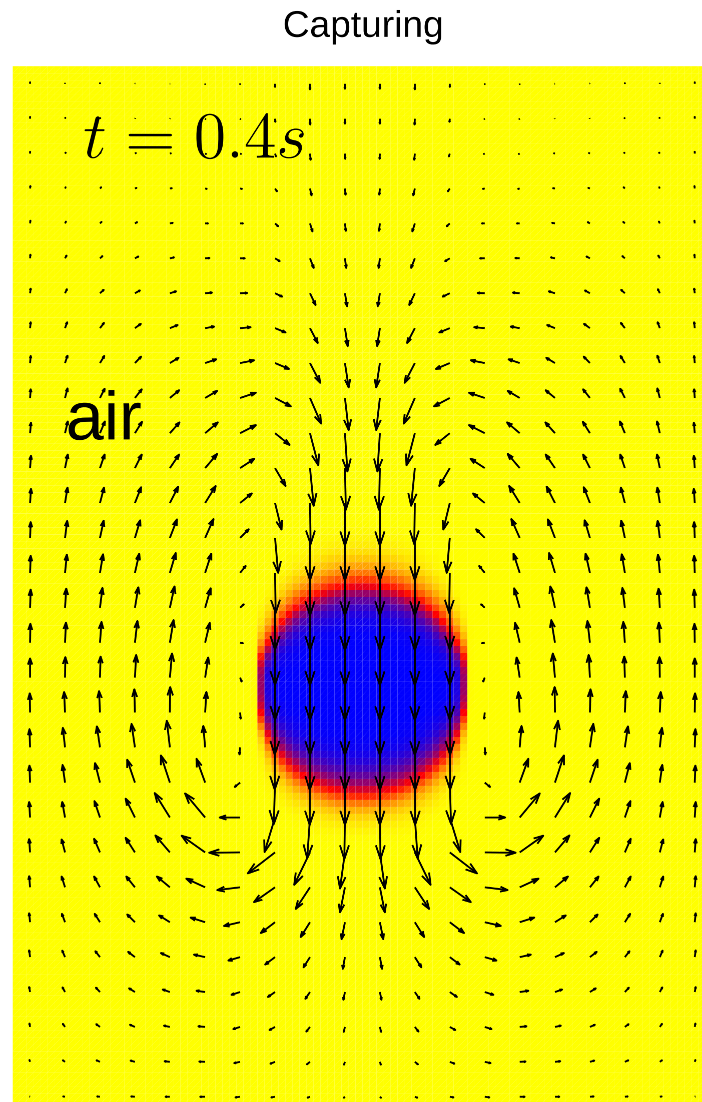

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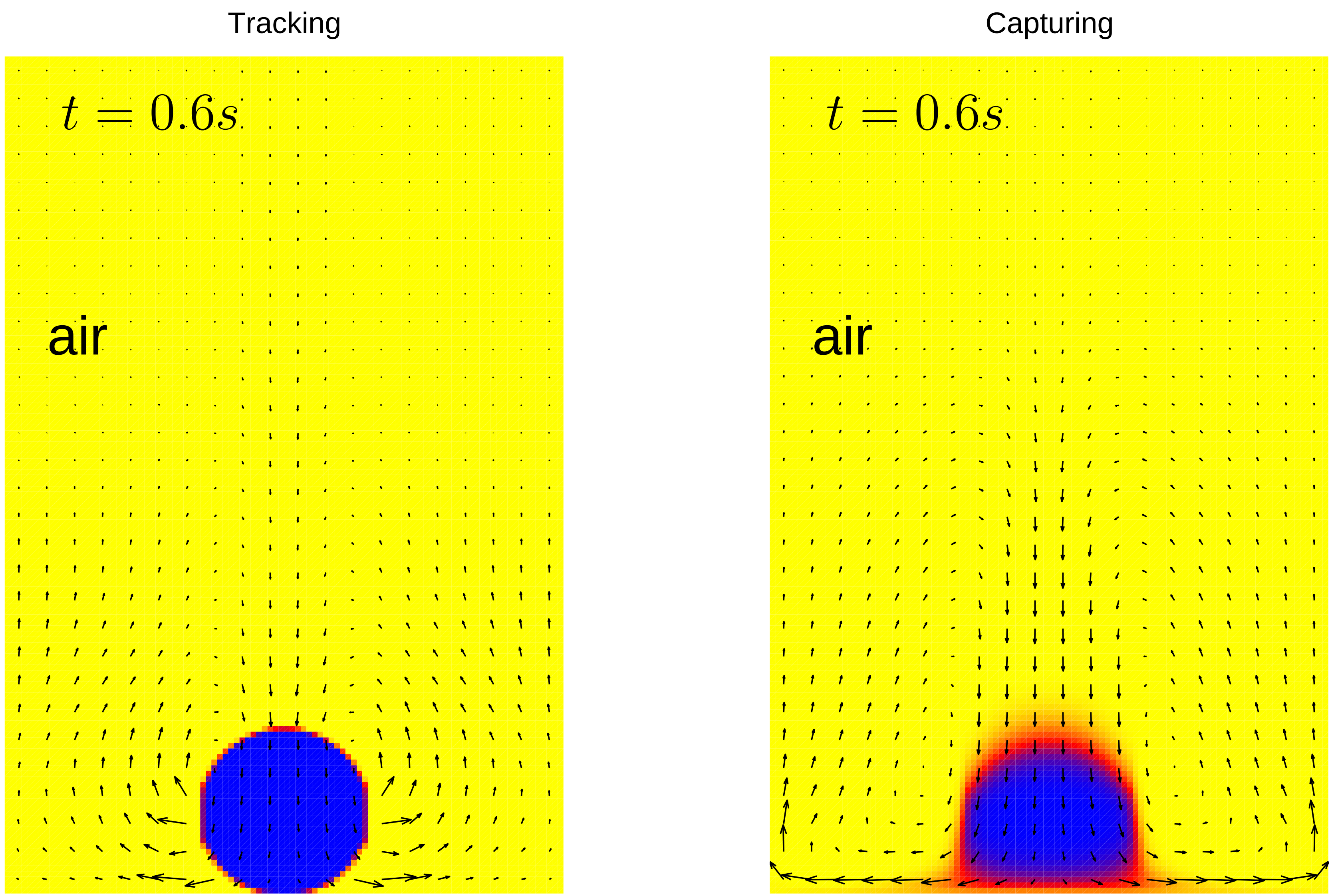

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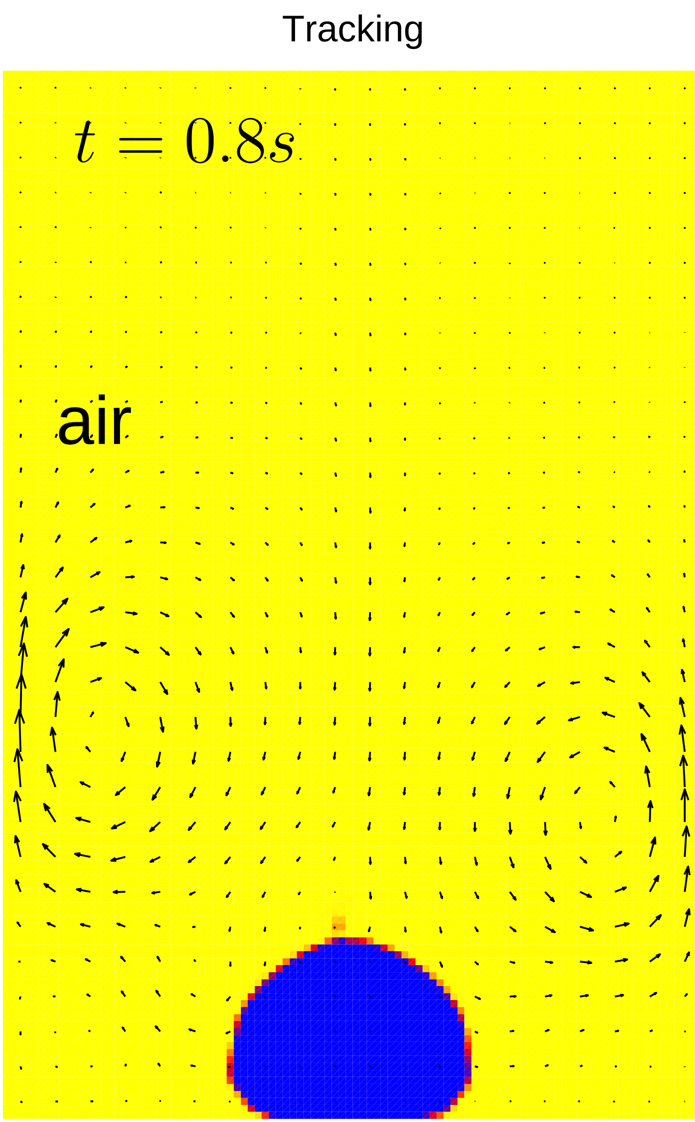
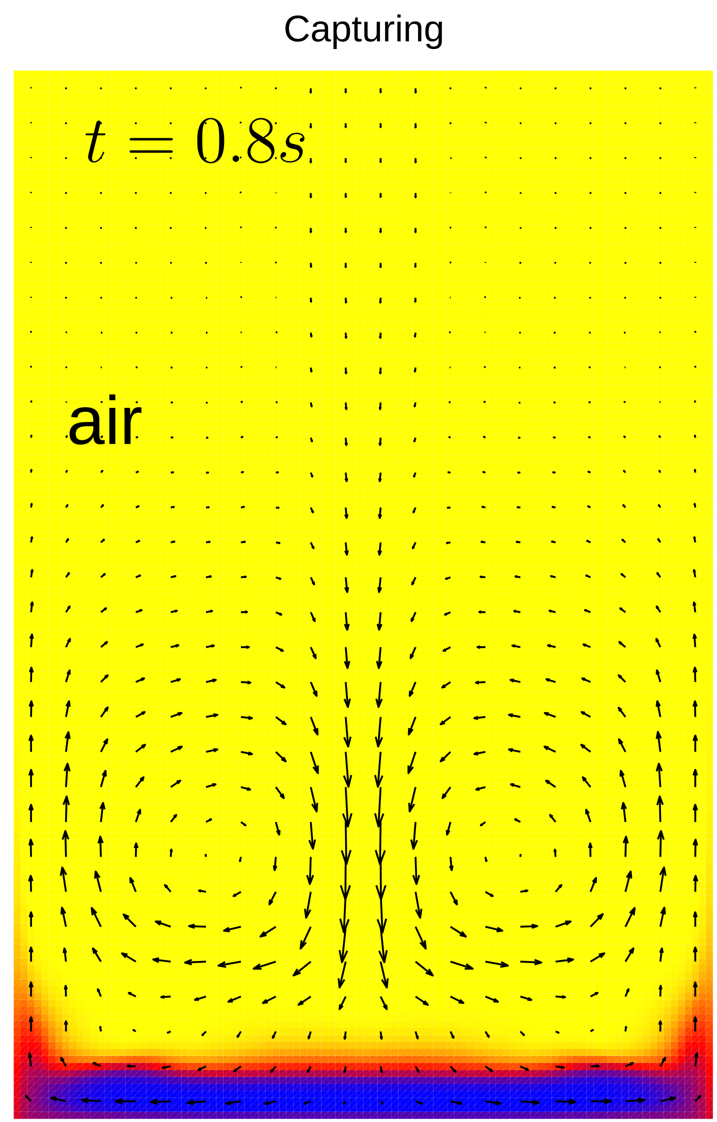
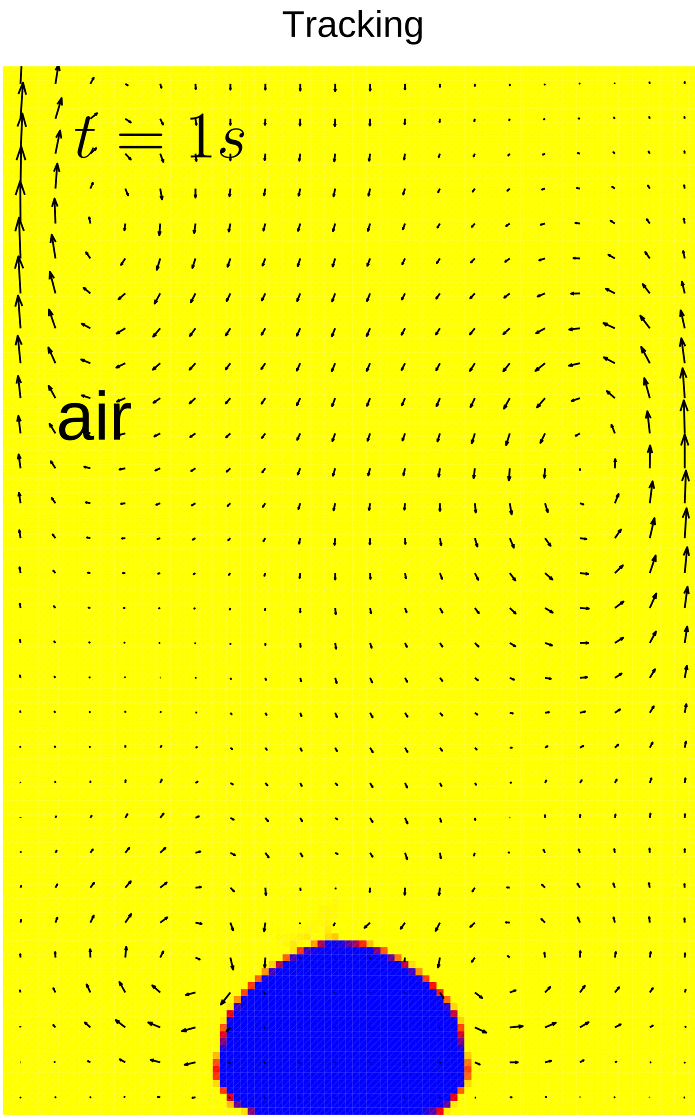
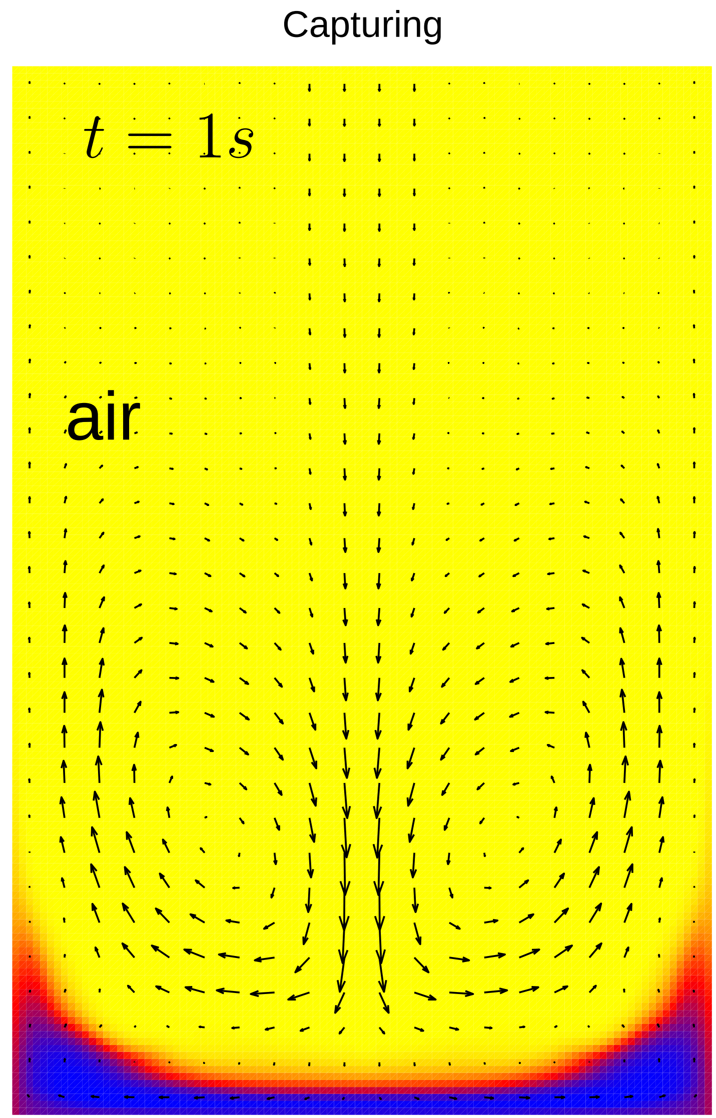




Liquid Drop Problem (Revisit)





Thank You

References

- (JCP 1998) An efficient shock-capturing algorithm for compressible multicomponent problems
- (JCP 1999, 2001) A fluid-mixture type algorithm for compressible multicomponent flow with van der Waals (Mie-Grüneisen) equation of state
- (JCP 2004) A fluid-mixture type algorithm for barotropic two-fluid flow Problems
- (JCP 2006) A wave-propagation based volume tracking method for compressible multicomponent flow in two space dimensions
- (Shock Waves 2006) A volume-fraction based algorithm for hybrid barotropic & non-barotropic two-fluid flow problems

Thermodynamic Stability

- Fundamental derivative of gas dynamics

$$\mathcal{G} = -\frac{V}{2}\frac{(\partial^2 p/\partial V^2)_S}{(\partial p/\partial V)_S}, \qquad S: \text{specific entropy}$$

- Assume fluid state satisfy $\mathcal{G} > 0$ for thermodynamic stability, *i.e.*,

$$(\partial^2 p/\partial V^2)_S > 0 \qquad \& \qquad (\partial p/\partial V)_S < 0$$

  - $(\partial^2 p/\partial V^2)_S > 0$ means convex EOS
  - $(\partial p/\partial V)_S < 0$ means real speed of sound, for

$$c^2 = \left(\frac{\partial p}{\partial \rho}\right)_S = -V^2\left(\frac{\partial p}{\partial V}\right)_S > 0$$

Homogeneous Flow Model (cont.)

- Mie-Grüneisen EOS: $p_k = p_{\text{ref}}(\rho_k) + \rho_k \Gamma(\rho_k)[e_k - e_{\text{ref}}(\rho_k)]$

$$\rho e = \sum_{k=1}^{2} \alpha_k \rho_k e_k = \sum_{k=1}^{2} \alpha_k \left[\frac{p - p_{\text{ref}}(\rho_k)}{\Gamma(\rho_k)} + \rho_k e_{\text{ref}}(\rho_k) \right] \quad \Longrightarrow$$

$$p = \left[\rho e - \sum_{k=1}^{2} \alpha_k \left(\frac{-p_{\text{ref}}(\rho_k)}{\Gamma(\rho_k)} + \rho_k e_{\text{ref}}(\rho_k) \right) \right]$$

Mie-Grüneisen Equations of State

- $(p_{\text{ref}}, e_{\text{ref}})$ lies along an isentrope
  1. Jones-Wilkins-Lee EOS for gaseous explosives

$$\Gamma(V) = \gamma - 1, \qquad V = 1/\rho$$

$$e_{\text{ref}}(V) = e_0 + \frac{\mathcal{A}\, V_0}{\mathcal{R}_1} \exp\left(\frac{-\mathcal{R}_1 V}{V_0}\right) + \frac{\mathcal{B}\, V_0}{\mathcal{R}_2} \exp\left(\frac{-\mathcal{R}_2 V}{V_0}\right)$$

$$p_{\text{ref}}(V) = p_0 + \mathcal{A}\ \exp\left(\frac{-\mathcal{R}_1 V}{V_0}\right) + \mathcal{B}\ \exp\left(\frac{-\mathcal{R}_2 V}{V_0}\right)$$

  2. Cochran-Chan EOS for solid explosives

$$\Gamma(V) = \gamma - 1$$

$$e_{\text{ref}}(V) = e_0 + \frac{-\mathcal{A}\, V_0}{1 - \mathcal{E}_1} \left[\left(\frac{V}{V_0}\right)^{1-\mathcal{E}_1} - 1\right] + \frac{\mathcal{B}\, V_0}{1 - \mathcal{E}_2} \left[\left(\frac{V}{V_0}\right)^{1-\mathcal{E}_2} - 1\right]$$

$$p_{\text{ref}}(V) = p_0 + \mathcal{A}\left(\frac{V}{V_0}\right)^{-\mathcal{E}_1} - \mathcal{B}\left(\frac{V}{V_0}\right)^{-\mathcal{E}_2}$$

Mie-Grüneisen EOS (cont.)

- $(p_{\text{ref}}, e_{\text{ref}})$ lies along a Hugoniot locus
  - Assume linear shock speed u_s & particle velocity u_p

$$u_s = c_0 + s\, u_p$$

  - We may derive the relations

$$\Gamma(V) = \Gamma_0 \left(\frac{V}{V_0}\right)^{\alpha}, \qquad \Gamma_0 = \gamma - 1$$

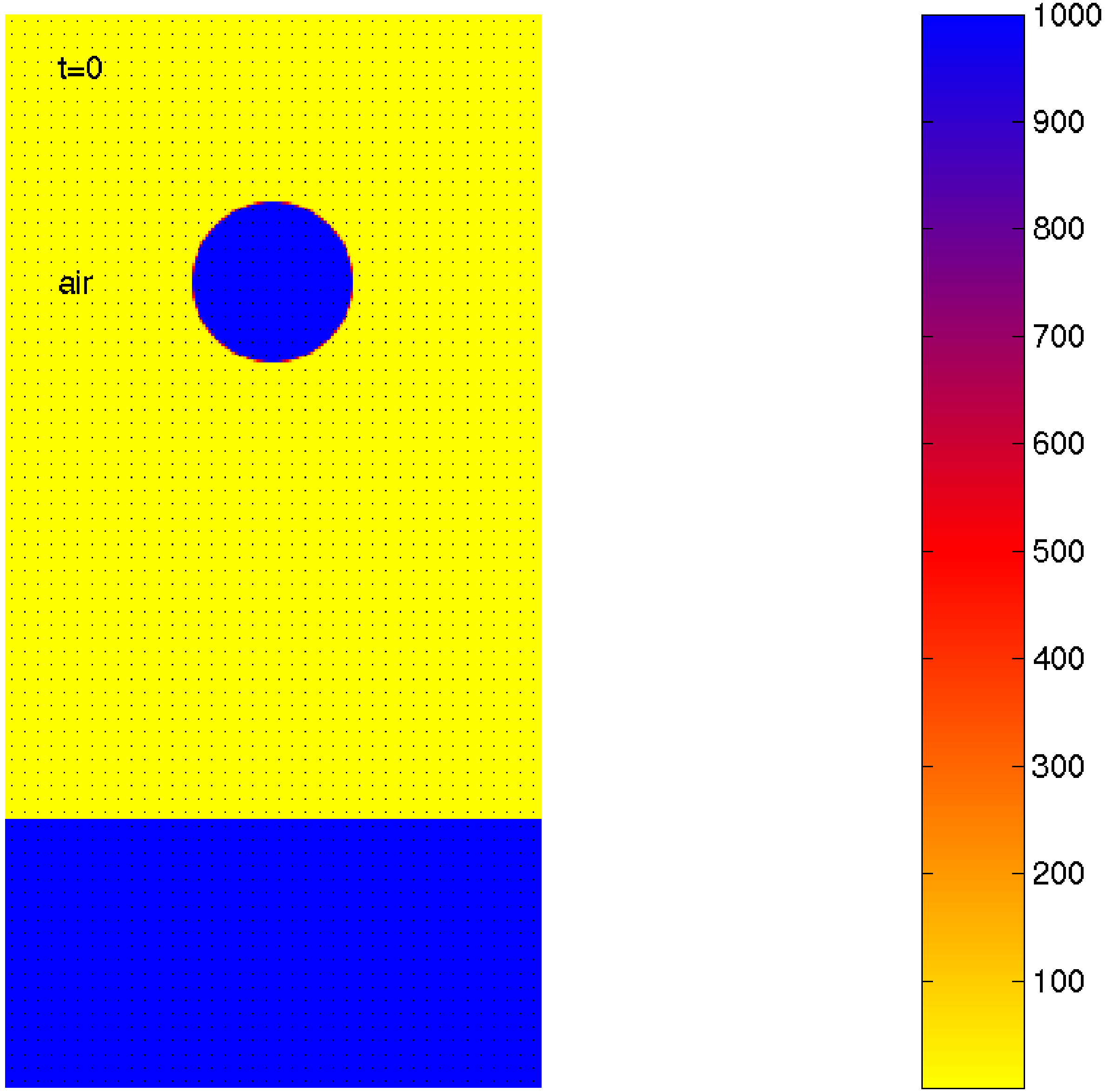
$$p_{\text{ref}}(V) = p_0 + \frac{{c_0}^2 (V_0 - V)}{[V_0 - s(V_0 - V)]^2}$$

$$e_{\text{ref}}(V) = e_0 + \frac{1}{2}\left[p_{\text{ref}}(V) + p_0\right](V_0 - V)$$

Material Quantities for Model EOS

JWL EOS	ρ_0(kg/m^3)	$\mathcal{A}$(GPa)	$\mathcal{B}$(GPa)	$\mathcal{R}_1$	$\mathcal{R}_2$	Γ
TNT1	1630	371.2	3.23	4.15	0.95	0.30
TNT2	1630	548.4	9.375	4.94	1.21	1.28
Water	1004	1582	-4.67	8.94	1.45	1.17
CC EOS	ρ_0(kg/m^3)	$\mathcal{A}$(GPa)	$\mathcal{B}$(GPa)	$\mathcal{E}_1$	$\mathcal{E}_2$	Γ
TNT	1840	12.87	13.42	4.1	3.1	0.93
Copper	8900	145.67	147.75	2.99	1.99	2
Shock EOS	ρ_0(kg/m^3)	c_0(m/s)	s	Γ_0	α	
Aluminum	2785	5328	1.338	2.0	1	
Copper	8924	3910	1.51	1.96	1	
Molybdenum	9961	4770	1.43	2.56	1	
MORB	2660	2100	1.68	1.18	1	
Water	1000	1483	2.0	2.0	10^{-4}	

Falling Liquid Drop Problem



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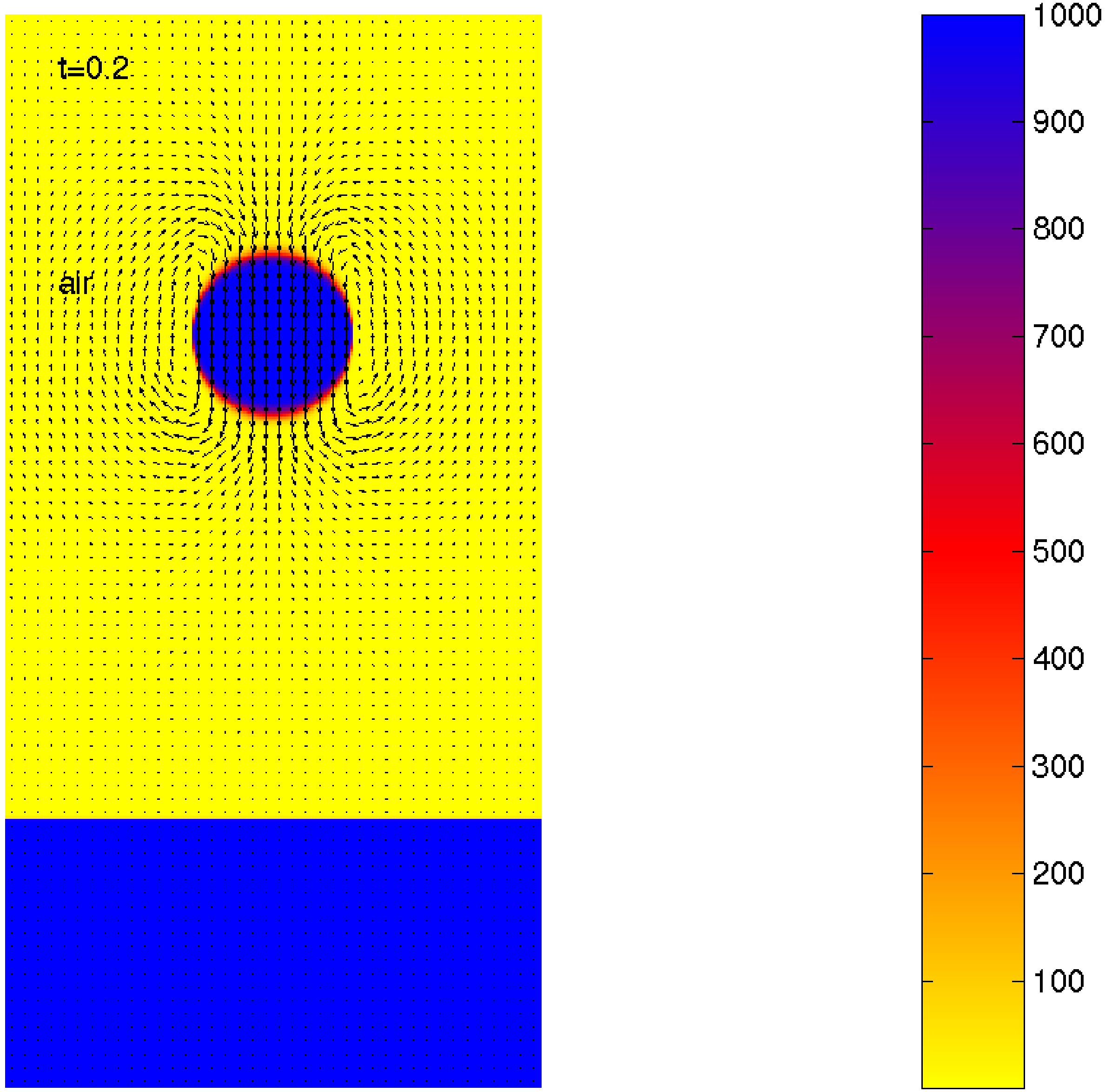

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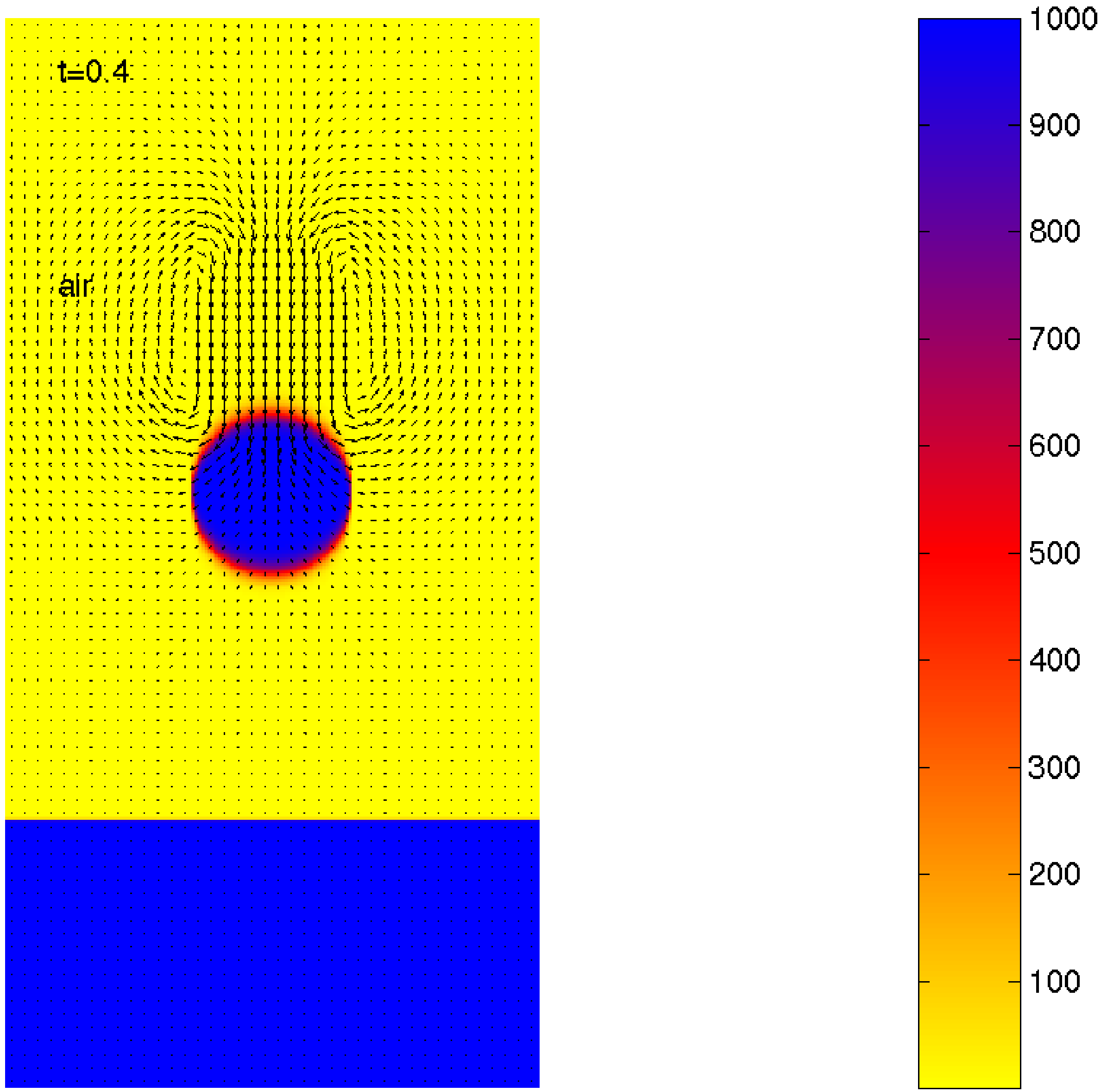

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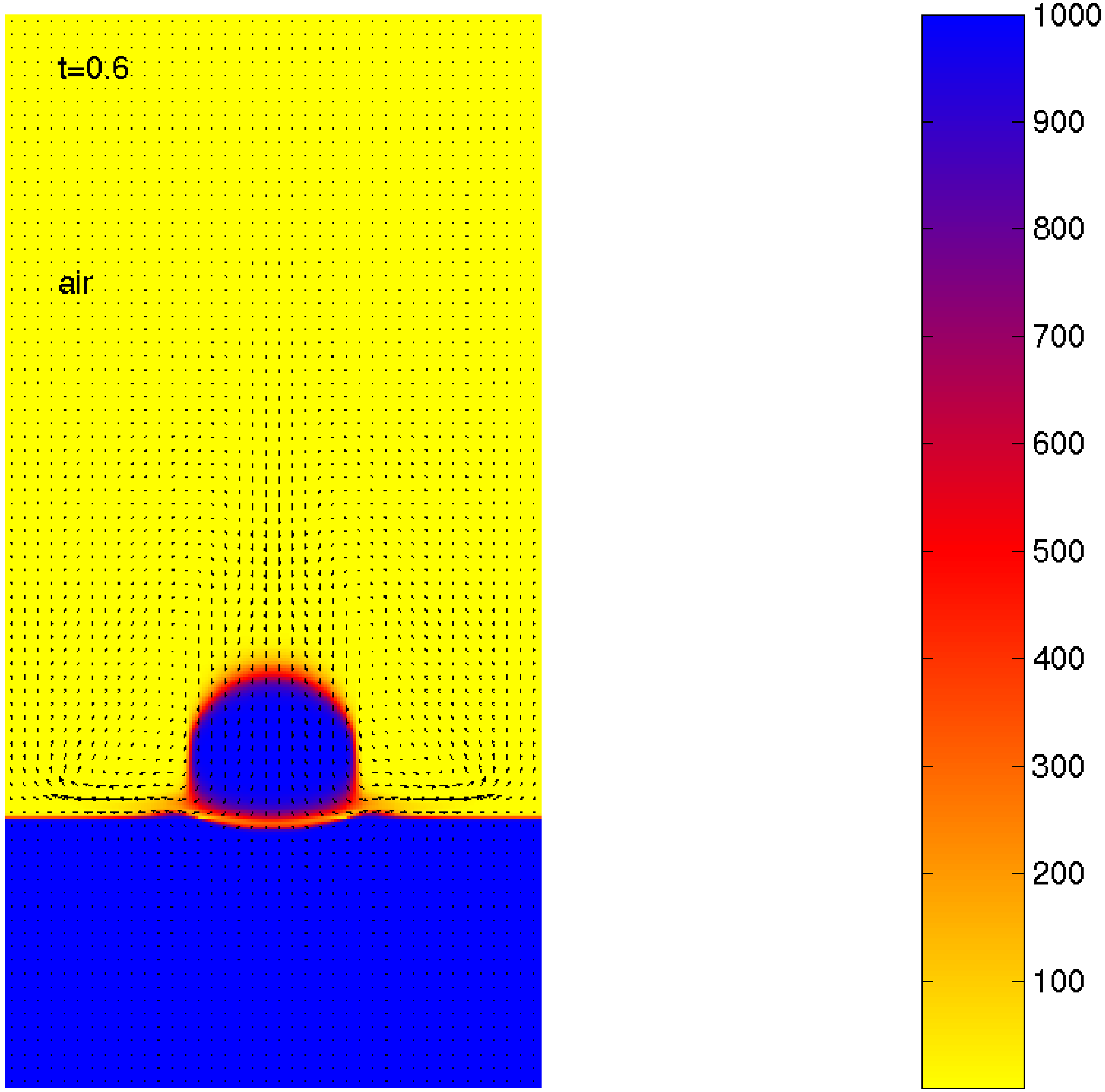

Falling Liquid Drop Problem