

Lab 3 (due 12/8)

This Lab Exercise is concerned with a nonlinear second-order differential equation of the form

$$\frac{d^2 y}{dt^2} + \alpha \frac{dy}{dt} + \beta y^+ + \gamma y^- = a + \lambda \sin(\mu t),$$

where it is assumed that $\beta > \gamma > 0$, $a > 0$, $\alpha > 0$, and

$$\frac{2\pi}{\sqrt{\beta}} < \frac{2\pi}{\mu} < \frac{\pi}{\sqrt{\beta}} + \frac{\pi}{\sqrt{\gamma}}.$$

Here we use the standard notations: $y^+ = \max(y, 0)$ and $y^- = \min(y, 0)$, in the equation. Note that this equation was introduced by Glover, Lazer, and McKenna to model the motion of a suspension bridge under various physical effects, such as gravity and wind, see the reference listed below for the details. Our aim is to study the existence and stability of large-scale oscillatory solutions of this model under realistic ranges of the parameters appearing in the model.

Now pick up a favorite numerical ODE solver of yours, say the “ode45” in Matlab or others, and perform the following tests with the fixed parameter values: $\alpha = 1/100$, $\beta = 17$, $\gamma = -13$, $a = 10$.

- (i) Do numerical experiment with $\mu = 4$, $y(0) = 10/17$ and $y'(0) = \lambda$ for $\lambda = 0.01, 0.06$, and 0.61 in large time, say after 1000 – 2000 time unit. Be sure to comment your results.
- (ii) Vary μ also, and investigate the solution behavior as a two-parameter $\lambda - \mu$ family.

Reference:

J. Glover, A. C. Lazer, and P. J. McKenna, Existence and stability of large scale nonlinear oscillations in suspension bridges, *Journal of Applied Mathematics and Physics (ZAMP)*, pp. 172-200, Vol. 40, March 1989.