作品評語

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The Four Color Map Problem was first proposed by Francis Guthrie (1952), a student in England. Trying to color the map of counties of England, he noticed that four different colors were sufficient in order that two countries sharing a boundary receiving different colors. He investigated many maps and raised The Four Color Map Problem. As not able to prove this fact, he asked his brother's teacher Augustus De Morgan at University College London. The problem then was spread out.

After more than a century, the problem was solved by Appel, Haken and Koch (1977) with the aids of computer, and it is known as The Four Color Theorem. The theorem now is often stated in term of the dual graph of a planar graph. It says that every planar graph is 4-colorable, that is, the vertices of a planar graph can be colored by 4 colors so that adjacent vertices receive different colors. Soon after then, Vizing (1976) and Erdős et al. (1979) independently generalized the concept of vertex coloring to list coloring. This article studies list coloring for planar graphs.

Voigt (1993) first gave examples of planar graphs that are not 4-choosable. On the other hand, Thomassen (1994) proved that every planar graph is 5-choosable. Hence, over the past decades, determining whether a planar graph is 4-choosable or not has received significant attention. Results of this lines are given by Lam et al. (1999), Wang and Lih (2001, 2002, 2002), Farzad (2009), Chen et al. (2016), Lin (2017) etc.

This article provides a new sufficient condition for a planar graph to be 4choosable. Different from the above research, in this article, each short cycle is allowed to exist. This article uses the Euler's polygon formula and the discharging method to prove the following interesting theorem.

Theorem: Every planar graph without adjacent 3-cycles, adjacent 4-cycles and starsuns is 4-choosable.

As conjectured at the end of the article, it is desirable to prove a stronger result by removing the condition of "without star-sun" from the above theorem.

Over all, this is a quit inspirited article.