

THE WORDS OF OMEGAS

機率思想的發展

台灣大學數學系謝南瑞

<http://www.math.ntu.edu.tw/~shiehnr>

****版權保留****

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- **Gauss and Laplace** 高斯與拉普拉斯的榮耀與 Kuso
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- Touch to Infinity 數學,科學,哲學,神學

From Alpha to Omega

隨機性在早期西方文明

- The Egyptians (1500 B.C.) 踝骨骰子
- The Greeks (500 B.C.) 神諭 (gods' wills) in “dog” and “venus”
- Jesus of Nazareth (the Christ, 0 B.C. --)
- The God's Will
- .. 你們要去使萬民作我的門徒…(太 28:19)

left: Fortuna (a goddess), right: an astragalus dice



Jesus the Christ



Light in the Dark Age

黑暗時代中的修道院與大學

- The collapse of Rome (455 A.D.)
- 教會, 修道院, 與 大學
- *The light shines in the darkness, but the darkness has not understood it (John 1:5)*
- Alcuin of York (735-804) and Charlemagne (742-814)
- Leonardo the Pisan (Fibonacci, 1170-1250)

from left to right: Fibonacci, Alcuin of York(middle), Charlemagne (left)



From Omega to Alpha

文藝復興：屬地智慧的發端

- The Renaissance 文藝復興
- With the Greeks there is on the whole little speculation about the natural phenomena of the earth (as opposed to the heavens) which is based on empirical hypotheses. With the renaissance of human intellect there is the beginning of the empirical tradition, from observation to hypothesis and back to observation again, which has led to the great strides of modern science (F.N. David 1962).
- 敬畏耶和華是知識的開端（箴言 1:7）
- 敬畏耶和華是智慧的開端（箴言 9:10）

Fermat and Pascal

費馬與巴斯卡的通信

- Pierre de Fermat (1601-1665)
- Blaise Pascal (1623-1662)
- Rev. Father M. Mersenne's "Academy", forerunner of Royal Society of London (1660) and Academie des Sciences de Paris (1665)
- In 1654, they have had a series of letters on "problem of points"
- 一般認為這是機率論的緣起,但
- 賭注分配問題先前已出現於 Fra Luca Paciolo (1445-1509)的 1494 書中

Pascal's letter to Fermat at July 29th 1654

- 甲乙兩人各出32元對賭, 先贏3局者拿全賭注
- 若未完成賭局應如何分賭本 ?

- 甲贏2局乙贏1局: 48 : 16
- 甲贏2局乙贏0局: 56 : 8
- 甲贏1局乙贏0局: 44 : 20

- Fermat, Pascal, Paciolo
(from left to right)



Gauss and Laplace

高斯與拉普拉斯的榮耀與 Kuso

- Carl Friedrich Gauss (1777-1855) 數學王子
- Pierre-Simon Laplace (1749-1827) 法國牛頓
-
- Gauss: theory of errors and method of least squares (1795,1809)
- Laplace: Analytic Theory of Probabilities (1812)
- 前者一般認為是統計學的發源
- 後者一般認為是機率論的大成(up to then)

- 但兩位都有 Kuso 名言：參考 E. T. Bell: Men of Mathematics (中譯: 大數學家, 九章)

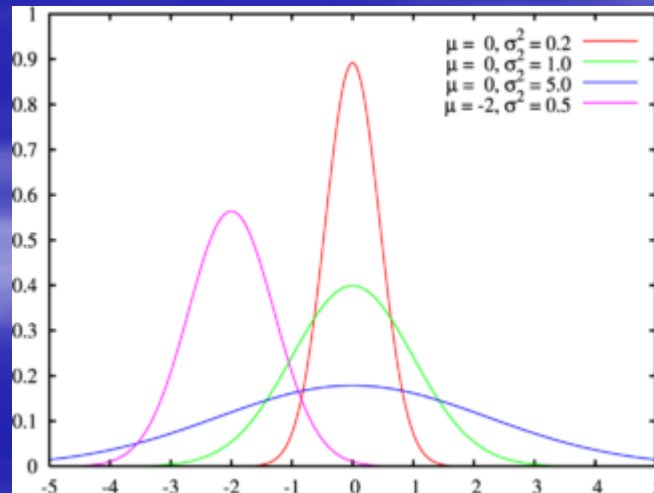
Gauss versus Cauchy Distribution

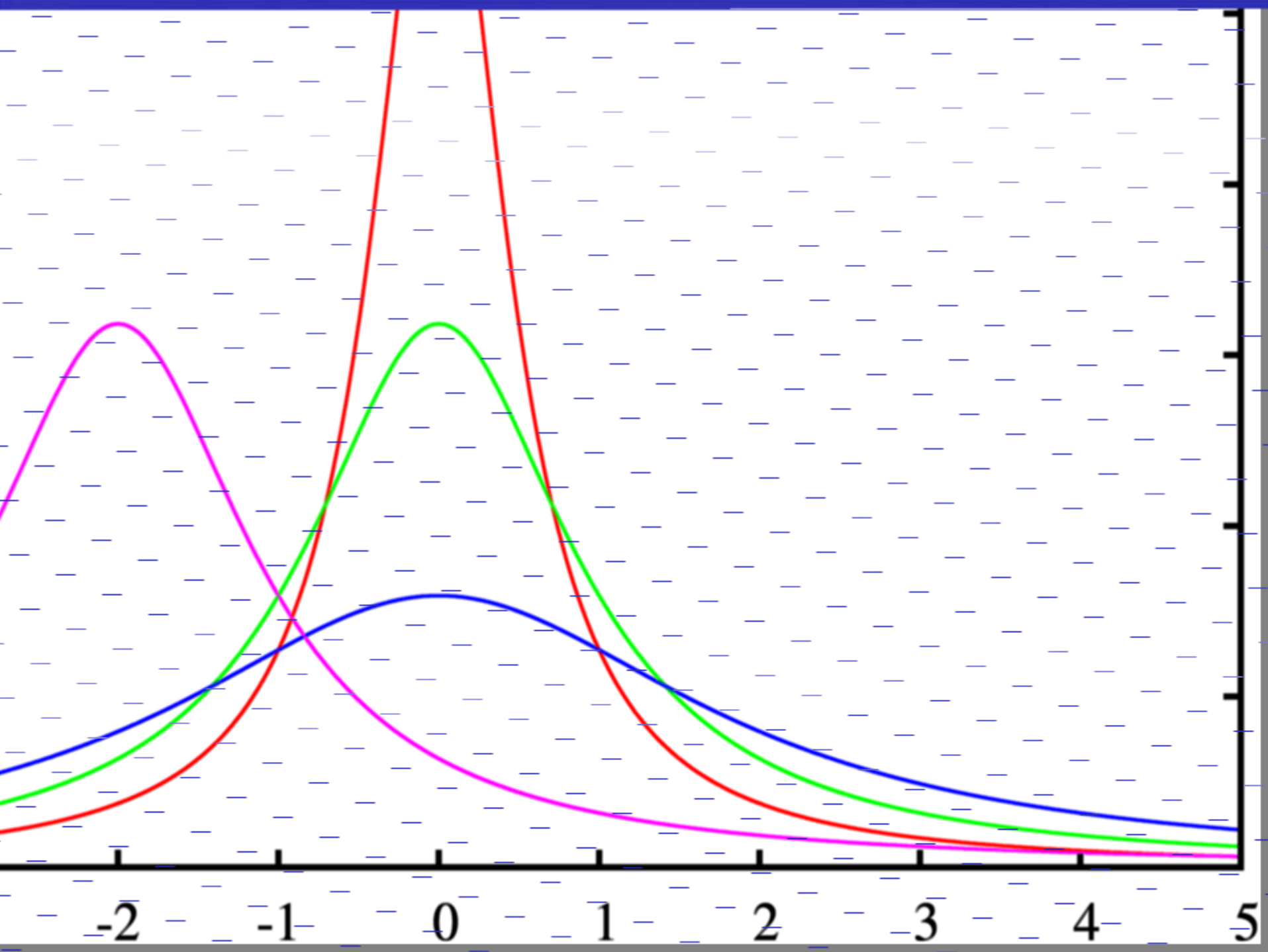
“正規” 與 “非正規”

- Augustin-Louis Cauchy (1789-1857)
- Gauss distribution (“normal” distribution)
- Cauchy distribution (“abnormal” distribution)
- probability density function, in standard form,
- $f_G(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$
- $f_C(x) = \frac{1}{\pi} \frac{1}{1+x^2}$
- 指數衰減 v.s. 厚尾現象

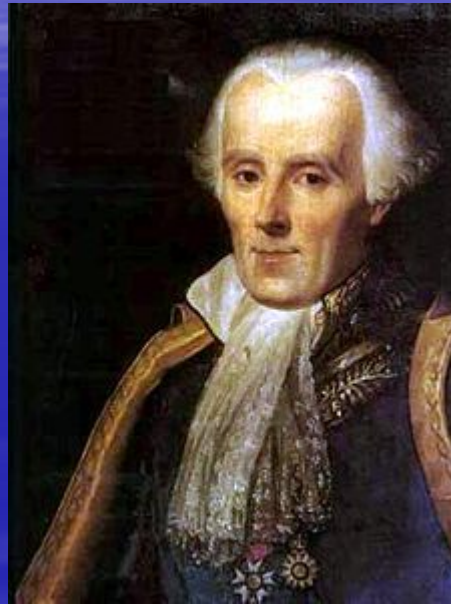
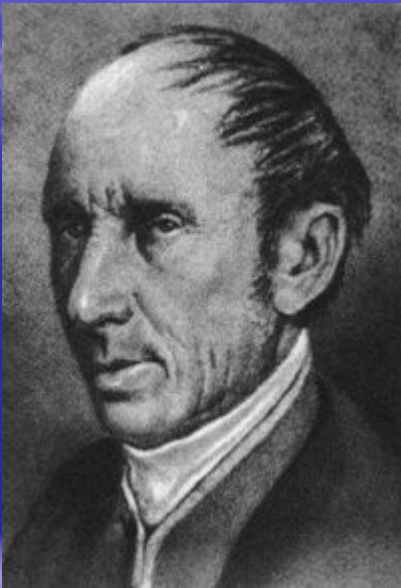
“Normal” distribution, correct and unbeatable?

- each of “n particles” contributes “1/n to the whole”, so that “the middle” plays normal and deciding role. Yet, it has been observed that “power law” and “heavy tail” phenomena appear both in earthly and heavenly sciences and wisdoms. Compare the curves below with those in the next slide





Cauchy, Gauss, Laplace, Poisson (from left to right)



Poisson and Poisson Process

泊松過程：等待

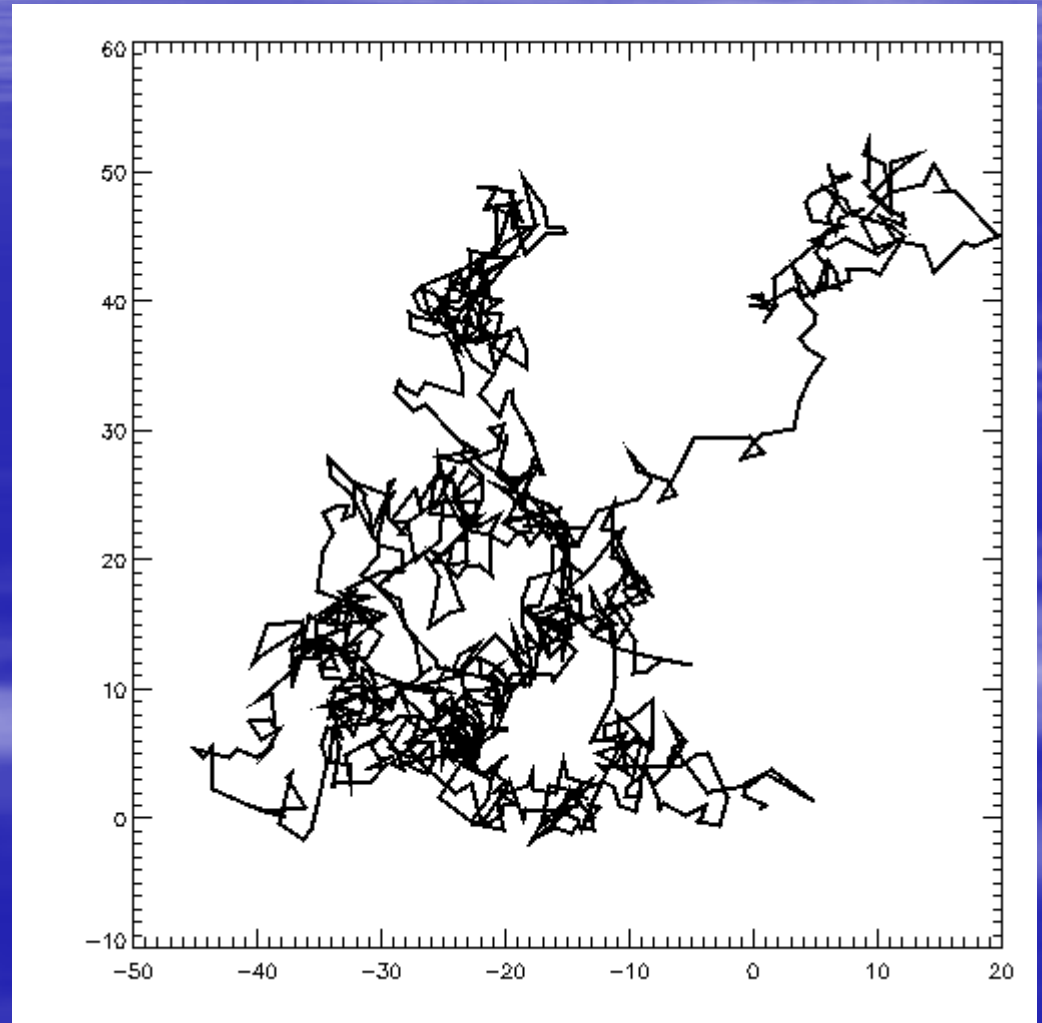
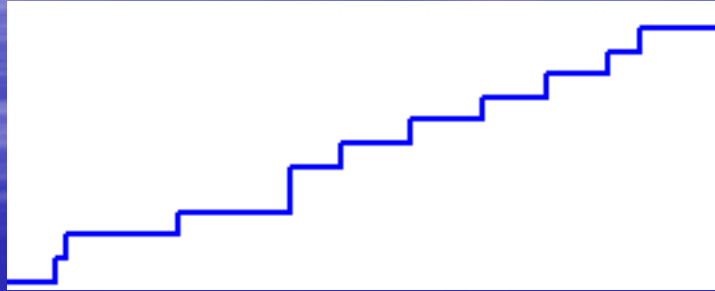
- Simeon-Denis Poisson (1781-1840)
- On the probability of the judgments of course of justice (1837)
- Poisson distribution : 以下為機率導論一考題
- Ms TW 打字錯誤個數視為某離散機率分佈. 已知平均而言每
- 20字中有1個錯字. 利用 CLT 估計 Ms TW 打 4500 字文章 錯字 < 200 個字之機率. 自行決定那個分佈.
- Poisson process : waiting for coming, 以下為隨機過程導論一考題
- Let X be the Poisson process with parameter λ . Let T_i , $i=1,2,\dots$ be the sequence of inter—arrival times, prove that it is iid with $\text{Exp}(\lambda)$ as common distribution. Derive the distribution of the total waiting time until n -th event, i.e. $S_n = T_1 + \dots + T_n$.

Brownian Motion and Wiener Process

布朗運動與維納過程：“生命力”與“分子互動”

- Robert Brown (1773-1858) 觀察
- Norbert Wiener (1894-1964) 建模
- 顯微鏡下花粉的運動
- 愛因斯坦的物理學家觀點(1906, Brownian motion and molecular movement), 一般將此與光電現象及相對論併列為愛因斯坦三大學術成就
-
- 愛因斯坦的 Kuso 名言：

left: Poisson path, right: Brown path



Mathematical Probability Theory

公理化機率論

- A.N. Kolmogorov (1903-1987)
- Foundation of Probability Theory (1933)
- The sample space Ω , is a nonempty set whose elements are known as outcomes or states of nature and are often given the symbol ω . The set of all the possible outcomes of an experiment is known as the sample space of the experiment.
- The probability measure P is a function from the sigma-algebra of events to the real numbers that assigns to each event a *probability* between 0 and 1. It must satisfy the probability axioms
- A random variable X is a measurable mapping from ω in Ω to $X(\omega)$ in \mathbb{R} (real number system).

■ THE WORDS OF OMEGAS

Probability in the 21st Century

- International Congress of Mathematicians,
- Madrid, Spain, 2006. Fields Medal is offered to those mathematicians *under 40*.
- Wendelin Werner is offered for his works based on Stochastic Loewner Evolutions.
- Additionally, Kiyosi Ito is offered the first Gauss Prize for his works on Stochastic Analysis.

Paradox in Probability

- Bertrand 諄(詭)論: 在圓周上隨機地取一弦, 弦長大於圓內接正三角形邊長的機率
- This problem was originally posed by Joseph Bertrand in his work, Calcul des probabilités (1888). Bertrand gave three arguments, all apparently valid, yet yielding inconsistent results.

機率論的迷思 I

- 以下引自 Richard von Mises: Probability, Statistics and Truth, Lecture One (1950, p. 9)
- We do not deal with the problem of the historical accuracy of Biblical narratives... Similarly we do not concern ourselves with any of those problems of the moral sciences The unlimited extension of the validity of the exact sciences was a characteristic feature of the exaggerated rationalism of the 18th century. We do not intend to commit the same mistake.

機率論的迷思 II

- 以下亦引自 Richard von Mises: Probability, Statistics and Truth, Lecture One (1950, p 17)
- Similarly, the insurance mathematicians (working for some insurance company)... use a figure such as the above 0.011 (company's death probability of a registered person with age 40 dies at 41)...It is utter nonsense to say, for instance, that Mr. X, now aged 40, has the probability 0.011 of dying in the course of the next year.

Muss es sein? Es muss sein!

條件機率

- Must it be ? It must be !
- 三門問題 Monty Hall Problem
- 0 – 1 律 (Kolmogorov, Hewitt-Savage)
- Decision Tree, 0 or 1 ?
- Philosophy of Probability

Probability in Complexity

- 以下引自 Thomas Lux: J. Econ. Beh. & Org. vol. 33 (1998) The socio-economic dynamics of speculative markets, p144.
- It has been highly debated, whether the apparent deviation of the (highly complicated) data from a random walk is due to stochastic or deterministic non-linearities.....
- 金融市場不是丟銅板, 想當然惡, 但數據的高複雜性由何而來?

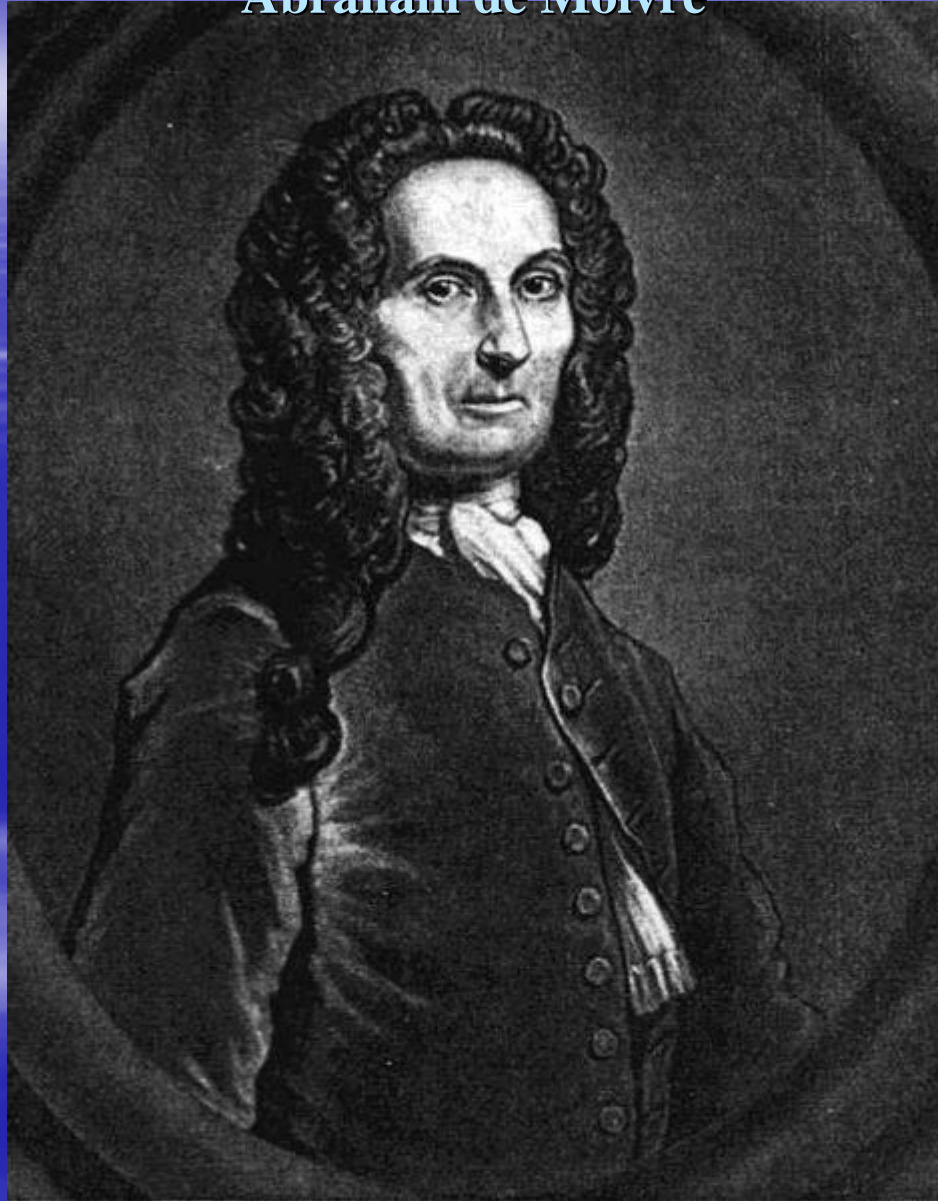
Job, Dirac, Heisenberg, Nicodemus

天問

Abraham de Moivre (隸美弗)

- 蒙福高壽 1667-1754 (牛頓 1643-1727)
- 因信仰流亡異國 (自1688 就 客居英國終生)
- 結識牛頓 受其賞識 獲選入英國皇家學會
- 找不到正式教職 以家教爲生
- 長期在一間咖啡店爲人計算賭注以補貼生活
- 計算得投銅板的中央極限定理
- 出版 *Doctrine of Chances* (1st ed 1718, 2nd ed 1738, 3rd ed 1756(posthumous))
- 在書的第三版卷末寫下信仰見證
- 書的內容被人標竊 書的學術榮耀被 Laplace 取代
- 但尚有 F. N. (Florence Nightingale) David 爲他 專章寫下
以上人生 供讀者低迴長思 (F.N. David: *Games, gods, and gambling, a history of probability and statistical ideas*, 1962, republished by Dover 1998)

Abraham de Moivre



Touch to Infinity

觸摸神

- 神賜予我們智慧與意志去探索祂
- 數學 (mathematics)
- 科學 (sciences)
- 哲學 (philosophy)
- 神學 (theology)
- 這麼個 from Alpha to Omega 的過程我們是否以
- 謙卑感恩底 Walk along with Him
- *For the Lord gives wisdom;*
- *from His mouth come knowledge and understanding*
- *(Proverbs 2:6)*