# 數學與工業

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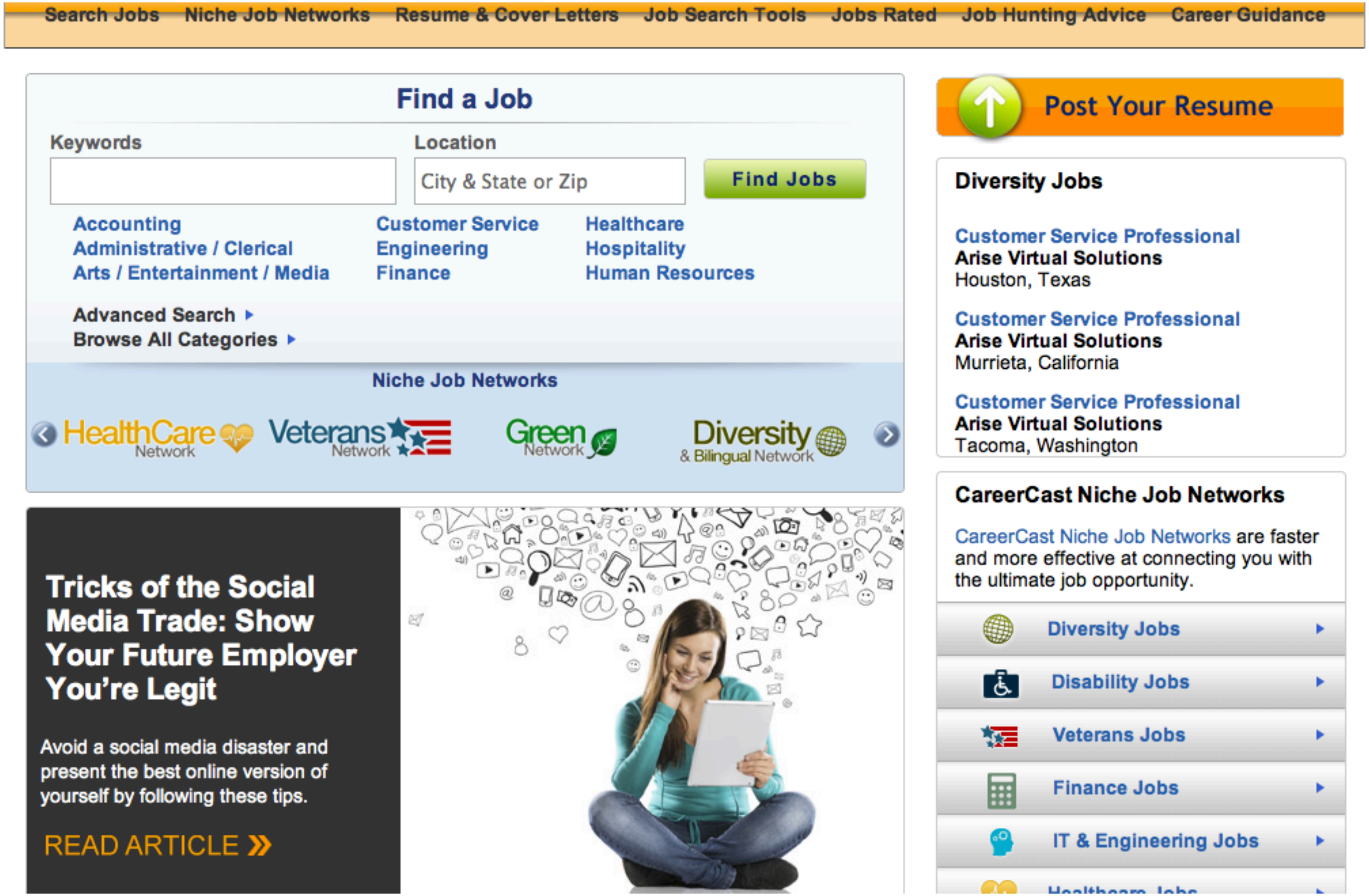
# Outline

- Part 1: 工業界需要數學嗎?
- Part 2: 工業界如何用數學
- Part 3: 如何做好準備

# Part 1: 工業界需要數學家嗎?

- Are mathematicians needed in Industry?
- SIAM report on Math in Industry

# CareerCast.com是美國的一個人力資源網站



# CareerCast 的一份報告
# Best Jobs in 2014

| 排名 | 職業 | 年收入（美金） | 2022 年就業成長率（%） |
|---|---|---|---|
| 1 | 數學家 | 101,360 | 23 |
| 2 | 大學教授 | 68,970 | 19 |
| 3 | 統計學家 | 75,560 | 27 |
| 4 | 精算師 | 93,680 | 26 |
| 5 | 聽覺矯正學家 | 69,720 | 34 |
| 6 | 齒科醫師 | 70,210 | 33 |
| 7 | 軟體工程師 | 93,350 | 22 |
| 8 | 電腦系統分析師 | 79,680 | 25 |
| 9 | 物理治療師 | 75,400 | 29 |
| 10 | 語言治療師 | 69,870 | 19 |



[轉載2014.4.18科技新報]

# CareerCast

- CareerCast 對 200 種行業排序是依據:
  - 收入
  - 工作環境
  - 工作壓力
  - 體能要求
  - 事業前景
- 使用勞工部與人口調查局的資料，並根據研究者的專業經驗。
- 註: 收入最高的行業仍是醫生與律師，但壓力與風險都較高。

# 其他相關報導

- US News 2014
  - Best business jobs: Market research analyst, operations research analyst
  - Best technology jobs: Software developer, computer systems analyst, web developer
- Business Insider
  - Software developer, computer system analyst -> data scientist
- Wall Street Journal 2014, 2013, …, 2009…
- search best jobs in 2014, you will find more…

- Mathematicians are needed in industry!

- 為什麼數學專業逐漸被重視?

# Who Hires Math Majors

- Math majors develop
  - analytical proficiencies,
  - knowledge of math theory and practical applications,
  - problem-solving skills
- that are highly valued by public and private companies in every industry, including
  - finance, computer and data science, and biotechnology.

# Who Hires Math Majors

- State Farm Insurance: 60,000 people
- Qualcomm: a wireless technology company: 150 offices
- Boeing: 170,000 people
- Google: 70 offices
- U.S. Government
- Intel
- Edward Jones: investment firm with 11,000 offices
- McKinsey & Company: leading employer of business analysts
- Boston Consulting Group
- Genentech: discovery and manufacturing of medicines.

# SIAM

- 美國的工業與應用數學會Society for Industrial and Applied Mathematics (SIAM) 於1950創立.
- 宗旨: 應用數學與計算科學是解決實際問題的關鍵學科。我們的任務是透過出版、研究與活動，建立數學與科技的合作橋樑。

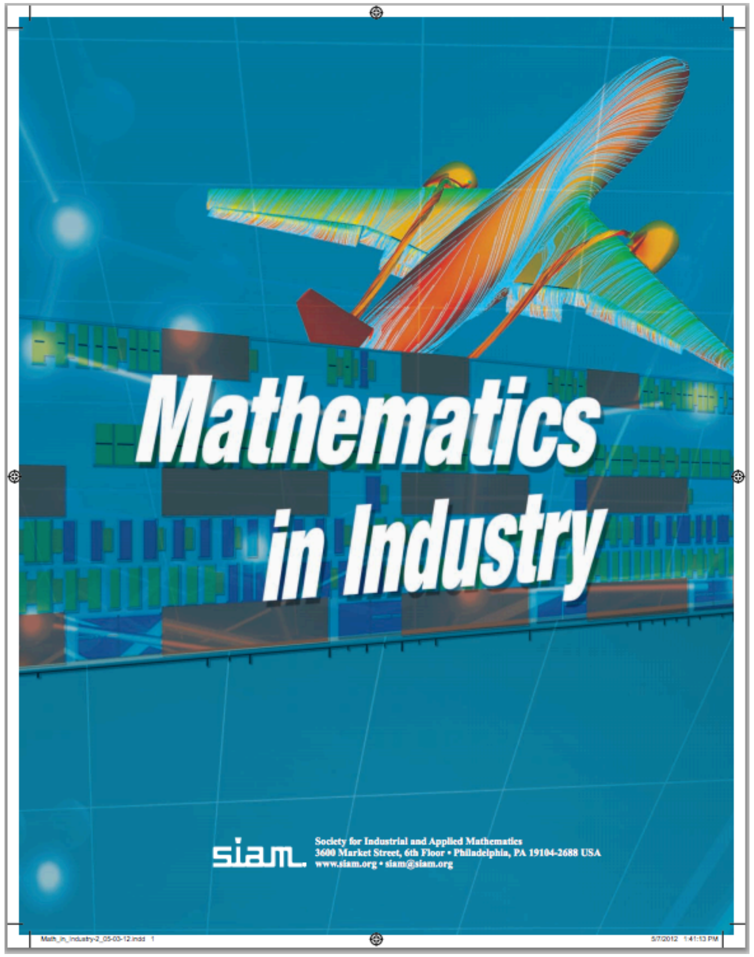
# SIAM Report on Mathematics In Industry
# MII 1996, MII 2012

- 檢視數學在學界外的角色
- 檢視數學家在非學界的工作環境
- 了解在業界數學家及其上司對其工作領域所需技能及背景知識的看法
- 提昇數學系研究生到業界工作機會的策略

# MII 2012: 一些新的挑戰

- 人類基因解碼與分子動力模擬開始用到藥物工業，需要懂得統計、資料探勘、模擬的人才。
- 金融界仍持續需要深入了解數學與金融的專家。
- 美國工業有許多由製造業轉到服務業，需要數學家提供各種諮詢服務

# SIAM Report on Mathematics in Industry (MII 2012)



# The role of mathematical sciences in Industry
# Trends and Case Studies



8種趨勢，18個範例

# 1. Business Analytics商務分析

- 數據引導的決策軟體(Data driven decision making)可以幫助許多公司在大量數據中找到有意義的規律。這就是所謂智慧型商務(Business intelligence)或商務分析(Business Analytics).
- 軟體工業賭上:數據引導的決策軟體將會是未來商業發展的走勢
- 主要的軟體高科技公司—IBM, Oracle(甲骨文), SAP(歐洲最大的軟體公司), Microsoft微軟—已投資$250 億美元在此領域. [Lohr, 2011-a]

# Business Analytics商務分析的內容

- 傳統部份: operations research (作業研究) 與 management science(管理科學).

- 新的元素: 巨量數據的應用

# 作業研究 與 管理科學

- 自1950年起，即已發展出以數學方法處理物流、倉儲、設施位址等優化問題
- 這些優化的技術，如線性規劃，對今日的工業仍十分重要。

# Data analysis for business (商務數據分析)

- 新的機會在發展 算法與技術，能以低價快速處理大量的數據。
- 公司可用智慧型商務與商務數據分析於市場行銷, 人力資源管理, 財務, 供應鏈管理, 設施位址, 風險管理, 產品與製造設計等。

# Business Insider Intelligence

- Business Insider 是美國商務與高科技新聞網站
  - 全球的網路用戶平均每天花費2.5個小時瀏覽社群網站，現在正是社群網站當道的時代，然而大家最關心的則是該如何有效運用這些網站背後的大數據，掌握社會趨勢並創造商機。
  - BI Intelligence的最新報告指出，只要觀察各大社群網站上的資訊與用戶行為就可以整理出有規律的數據，而如果能有效的掌握社群網站背後的大數據，則可以針對不同網站擬定策略，達到跨社群媒體行銷的第一步。

(來源:Business Next數位時代譯自BI Intellengence)

# 商務分析背後的人工智慧

- 《危險邊緣》(Jeopardy)是由梅夫·格里芬在1964年創建的美國的電視智力競賽節目。就像同一類的其它節目，節目涵蓋了歷史、語言、文學、藝術、科技、流行文化、體育、地理、文字遊戲等多方面內容。
- 2011年2月14日的《危險邊緣》節目中，IBM華生超級電腦擊敗人腦。
- IBM華生超級電腦有強大的自然語言的翻譯解釋能力、能處理無結構的資料、並作深度分析(deep analysis)。這些成果已可用在許多科學、健保、金融服務、及許多工業上的的問題 [Groenfeldt, 2011] 。
- 人工智慧涉及許多優化、統計、計算方面的問題。

# 商務分析背後的
# 高性能計算，雲端計算

- 許多行業有興趣於高性能計算來解決其工業問題
- 工業界需要數學建模、程式設計、數學軟體，並能在平行的計算平台進行模擬。
- 許多公司並非自己擁有平行電腦，而是使用雲端計算。

# 2. 跨學科設計優化 與 電腦輔助設計

Multidisciplinary Design Optimization(MDO) and
Computer Aided Design (CAD)

Boeing 787 landing at the end of its maiden flight, illustrating the flexibility of its wings.

# MDO and CAD

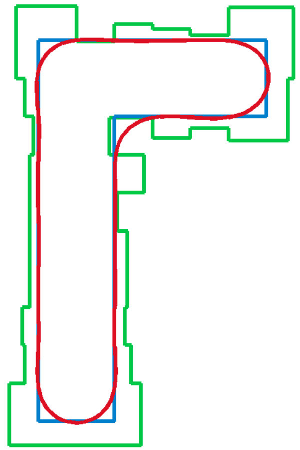
- 2011年10月Boeing 787 夢想航空號首航由東京飛到香港。
- 波音787使用光纖強化塑料，比鋁材料有更佳的強度-重量比，可省20%燃油。
- 波音787窗戶較大，卻可承受較高的氣壓差。

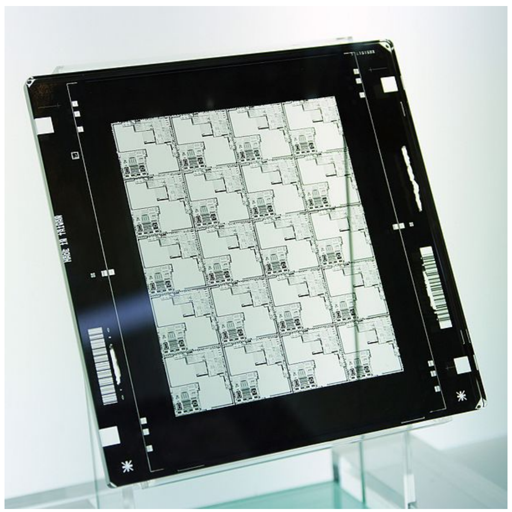
# MDO and CAD

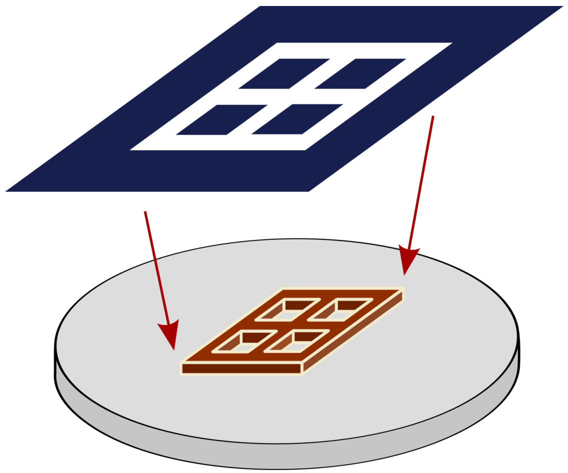
- 787 完全在電腦上設計.
- 787飛行時機翼彎曲可達3公尺，電腦要整合流體力學與結構力學作最佳化設計。
- 787內部設計可以應客戶需求，直接設計並直接連上生產線。
- 設計所需數學工具包括 computational linear algebra, differential equations, operation research, computational geometry, optimization, optimal control, data management, and a variety of statistical techniques。

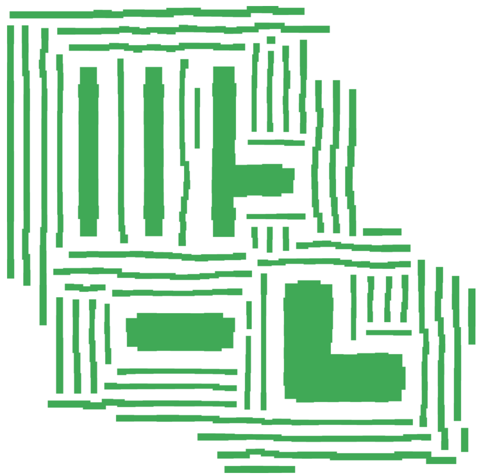
# 其他優化設計的例子 1

- 光罩設計:用於半導體製成
- 光罩設計要解複雜的反問題



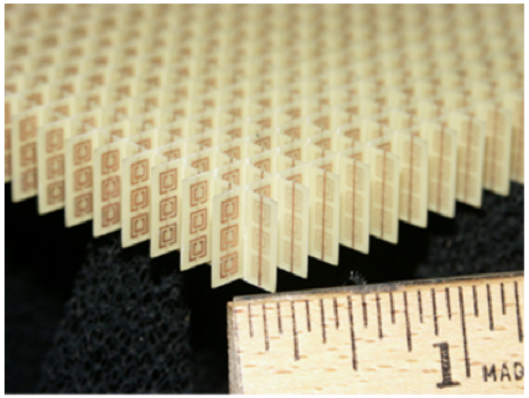


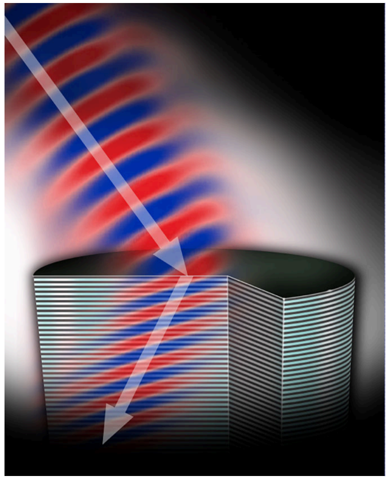




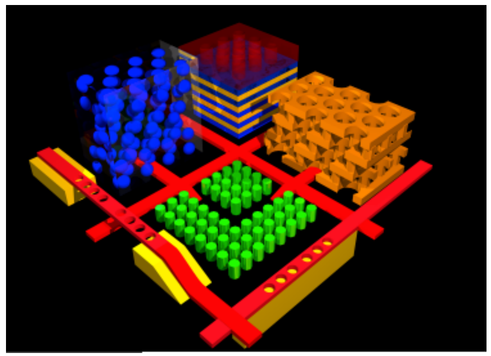
# 其他優化設計的例子 2

- 新穎材料的設計: 要解Maxwell方程的反問題 負折射係數，做隱形材料。





- 光子晶體的設計要解反問題
  - 濾波材料



# 3. 快速成長的IT領域

- Computer vision (電腦視覺)
- Image processing (影像處理)
- Imaging(造影)
- natural language processing( 自然語言處理)
- information retrieval (信息重獲)
- machine learning (機器學習)

也需要大量的數學

# 比如: Image Analysis and Data Mining

- Science Applications International Corporation (SAIC) 是一家承包美國國防部計畫的公司
- SAIC 發展了一套軍事用的智慧型偵查監測系統(ISR) 稱作AIMES
- AIMES分析無人駕駛飛機所錄之龐大的動態影像資料，可區別靜態背景與動態物件，可針對動態物件放大。AIMS為可攜式，可直接在戰場上部署。
- AIMES也可用於民用的防災或犯罪偵防，以及機械運轉偵測。
- AIMES裡的影像分析用到大量的數學，是當前熱門的研究課題。

# Part 2 工業界怎麼用數學
# How math is used in Industry?

- How google works
- How Amazon recommends books
- How to reduce imaging time in MRI
- How to denoise a corrupted picture
- How to inpaint a corrupted painting
- Some other info on ``math in industry.''

# How google works

- 爬挖資料 (Crawling)
- 建立索引 (Indexing) 60 trillion pages
- 排序(Search algorithm and page ranking)

# How google works

Three steps to make search engine work [1]

- Googlebot: Google's web crawling robot
  It crawls, downloads and fetches webpages in the cyberworld to the indexer.
- Indexer: sorts and indexes every word on every page, and stores them in a huge database
- Query processor: perform page ranking algorithm upon your query

1. Lars Elden, Matrix Methods in Data Mining and Pattern Recognition, SIAM
2. PageRank, Wiki
3. GoogleGuide, how google works

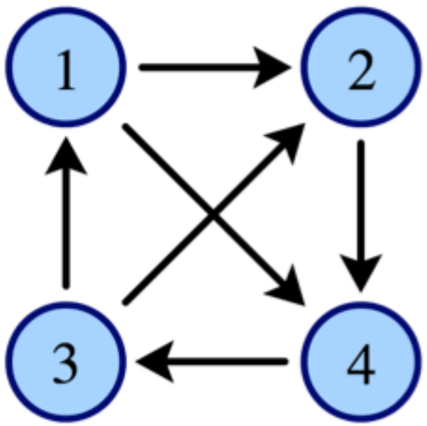
# Query Processor--PageRank

- It is a link analysis of webpages
- It was first suggested by Gabriel Pinski and Francis Narin in 1976 for ranking scientific journals.
- Two Stanford graduate students Sergey Brin and Larry Page developed PageRank as a new kind of search engine in 1996. They founded Google Inc. in 1998. PageRank is the basis of google's search tools.

# PageRank: A link analysis

- Each page (with inquiry words) a node of a graph and is ordered from $1, \dots, n$
- $N_j$: the number of pages that $j$ is linked to (out-link).
- Define $Q_{ij} = 1/N_j$ if there is a link from $j$ to $i$.

$$\sum_i Q_{ij} = 1.$$




$$Q = \begin{bmatrix} 0 & 0 & 1/2 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \\ 1/2 & 1 & 0 & 0 \end{bmatrix}$$

- Weight $r_i \geq 0$: the importance of page $i$ to the inquiry.
- Normalized by $\sum_j r_j = 1$.
- The importance $r_i$ is weighted by

$$r_i = \sum_j \frac{r_j}{N_j} = \sum_j Q_{ij} r_j.$$

- It is an eigenvalue problem!

$$\boxed{Qr = r.}$$

# Mathematical Theory

- Markov chain, Random walk
- Perron-Frobenius Theorem
- Directed Graph Theory

# A random walk interpretation

- $(r_1, \cdots, r_n)^T$ is a distribution of the importance of each page for a particular inquiry.
- Start from, say $r^{(0)} = (\frac{1}{n}, \cdots, \frac{1}{n})^T$, perform $Qr^0$.
- We can continue this process

  $$r^{(n+1)} = Qr^{(n)},$$

  This is called a random walk.
- Hopefully $r^{(n)}$ converges to $r$ with

  $$Qr = r.$$

  If so, then the ranking is determined by the order of magnitudes of $r_i$.

# Two problems

- We may get stuck at some page
- The random walk may not converge

- If column $j$ is zero, which means that $j$ does not link to any other page. To avoid the random walk gets stuck at page $j$, we modify $Q$ to $P$ by: if the $j$th column is zero, replace this column by $e/N$, where $e := (1, \cdots, 1)^T$.
- $P$ is called a stochastic matrix, that is $\sum_i p_{ij} = 1$.
- Find the eigenvalue of $P$ with largest eigenvalue $1$
- It does not change the order of $r$?!

$$Q = \begin{bmatrix} 0 & 1/3 & 0 & 0 & 1/2 & 1/2 \\ 1/3 & 0 & 0 & 0 & 0 & 0 \\ 1/3 & 1/3 & 0 & 0 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 1/3 & 0 & 1/2 & 0 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 1/3 & 0 & 1/6 & 1/2 & 1/2 \\ 1/3 & 0 & 0 & 1/6 & 0 & 0 \\ 1/3 & 1/3 & 0 & 1/6 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 1/6 & 0 & 0 \\ 0 & 0 & 1/3 & 1/6 & 0 & 1/2 \\ 0 & 0 & 1/3 & 1/6 & 1/2 & 0 \end{bmatrix}$$

# Perron-Frobenius Theorem

**Definition**

- A matrix $A$ is called reducible if there is a permutation $P$ such that
$$PAP^T = \begin{pmatrix} X & Y \\ 0 & Z \end{pmatrix}.$$
- We say $A > 0$ if all $a_{ij} > 0$.

**Theorem (Perron-Frobenius)**

*If $A > 0$ is an irreducible stochastic matrix, then there exists a unique $r$ such that $Ar = r$. All other eigenvalues are strictly less than 1.*

# Perturbation

**Theorem**

*Let $P$ be a stochastic matrix. Let $0 < \epsilon < 1$. Define*

$$A = (1 - \epsilon)P + \epsilon \frac{1}{N} e e^T.$$

*Then $A > 0$ is an irreducible stochastic matrix. Thus, there exists a unique $r$ such that $Ar = r$. Moreover, let*

$$r^{(n+1)} := A r^{(n)},$$

$$r^{(0)} > 0, |r^{(0)}|_1 = 1,$$

*then $r^{(n)} \to r$.*

# Challenges

- 60 Trillions of individual pages
- Size of stochastic matrix can be hundreds of thousands
- Fast algorithm

# Part 2 工業界怎麼用數學
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# How Amazon recommends books

- 亞馬遜網上書店 Amazon.com 是美國最大的一家網路電子商務公司，是網路上最早開始經營電子商務的公司之一，亞馬遜書店成立於1995年，一開始只經營網路的書籍銷售業務，現在則擴及了範圍相當廣的其他產品，包括了DVD、音樂光碟、電腦、軟體、電視遊戲、電子產品、衣服、傢具等等。
- 亞馬遜會根據你購買或評比過的書籍，推薦你相關的書籍

# Recommender Systems

Recommender Systems: An Introduction - Amazon.com
www.amazon.com › ... › Expert Systems ▾ Amazon.com ▾
Recommender Systems: An Introduction [Dietmar Jannach, Markus Zanker, Alexander Felfernig, Gerhard Friedrich] on Amazon.com. *FREE* shipping on ...

Introduction to Recommender Systems | Coursera

https://www.coursera.org/course/recsys ▾ Coursera ▾
Introduction to Recommender Systems is a free online class taught by Joseph A Konstan and Michael D ...

Recommender system - Wikipedia, the free encyclopedia
en.wikipedia.org/wiki/Recommender_system ▾ Wikipedia ▾
Recommender systems or recommendation systems (sometimes replacing ..... a strong culture of data privacy and every attempt to introduce any level of user ...

Recommender Systems: An Introduction
www.recommenderbook.net/recommender-systems-introduction ▾
Website for the book 'Recommender Systems: An Introduction'


The next few slides are copied from ``Recommender System, An Introduction,''

# Recommender systems

- Collaborative filtering
- Content-based filtering
- Knowledge-based recommendations
- Hybrid recommender systems

# Collaborative Filtering

- Given (incomplete) data of rating
- Determine whether Albert will like or dislike item 5

| | Item1 | Item2 | Item3 | Item4 | Item5 |
|---|---|---|---|---|---|
| Albert | 3 | 4 | 5 | 3 | ? |
| User1 | 5 | 2 | 4 | 4 | 3 |
| User2 | 3 | 2 | 2 | 1 | 5 |
| User3 | 2 | 5 | 5 | 3 | 4 |
| User4 | 4 | 1 | 3 | 4 | 2 |

# Rating by correlation-weights

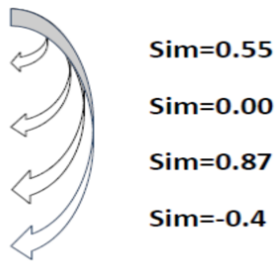
- Define similarity between two user $a$, $b$ by the correlation

$$\text{sim}(a,b) = \frac{\sum_p (r_{a,p} - \bar{r}_a)(r_{b,p} - \bar{r}_b)}{\sqrt{\sum_p (r_{a,p} - \bar{r}_a)^2} \sqrt{\sum_p (r_{b,p} - \bar{r}_b)^2}}$$

- Define prediction of rating of user $a$ on item $p$ by

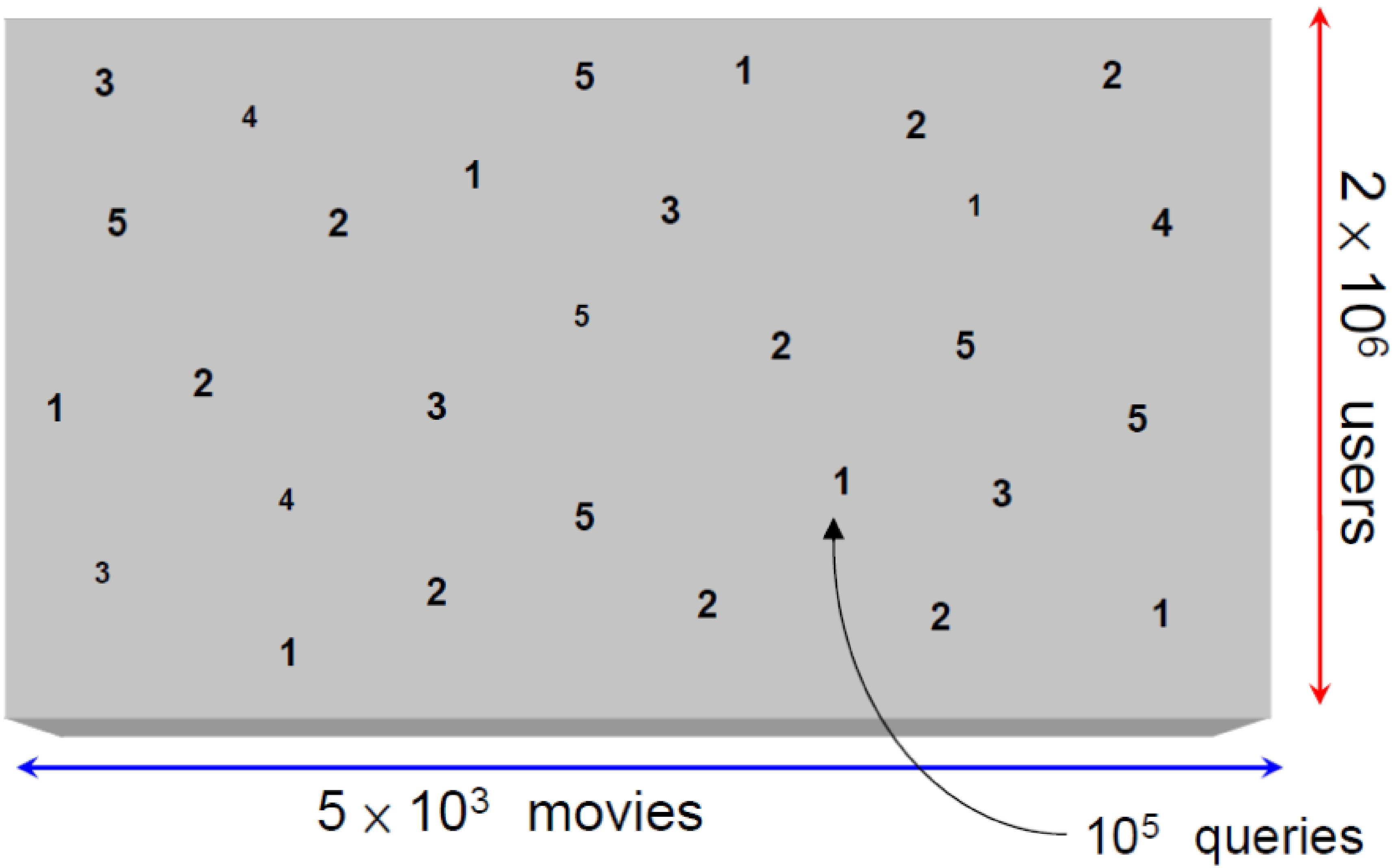
$$\text{pred}(a,p) = \bar{r}_a + \frac{\sum_{b \in N} \text{sim}(a,b) * (r_{b,p} - \bar{r}_b)}{\sum_{b \in N} \text{sim}(a,b)}$$

| | Item1 | Item2 | Item3 | Item4 | Item5 |
|---|---|---|---|---|---|
| Albert | 3 | 4 | 5 | 3 | ? |
| User1 | 5 | 2 | 4 | 4 | 3 |
| User2 | 3 | 2 | 2 | 1 | 5 |
| User3 | 2 | 5 | 5 | 3 | 4 |
| User4 | 4 | 1 | 3 | 4 | 2 |




Reproduce from Powerpoint-Slides for Recommender Systems - An Introduction

# Another approach: Matrix completion




Reproduce from Sewoong Oh, Standard University 2010

# Netflix Problem

- Netflix是一家美國公司，提供北美地區綫上播放DVD的出租業務。
- 2011年4月，Netflix宣布在美有2360萬用戶，而全世界則超過2600萬，可提供10萬種DVD選擇。
- Netflix會根據用戶訂閱過或評比過的資料提供推薦影片。

# Netflix獎

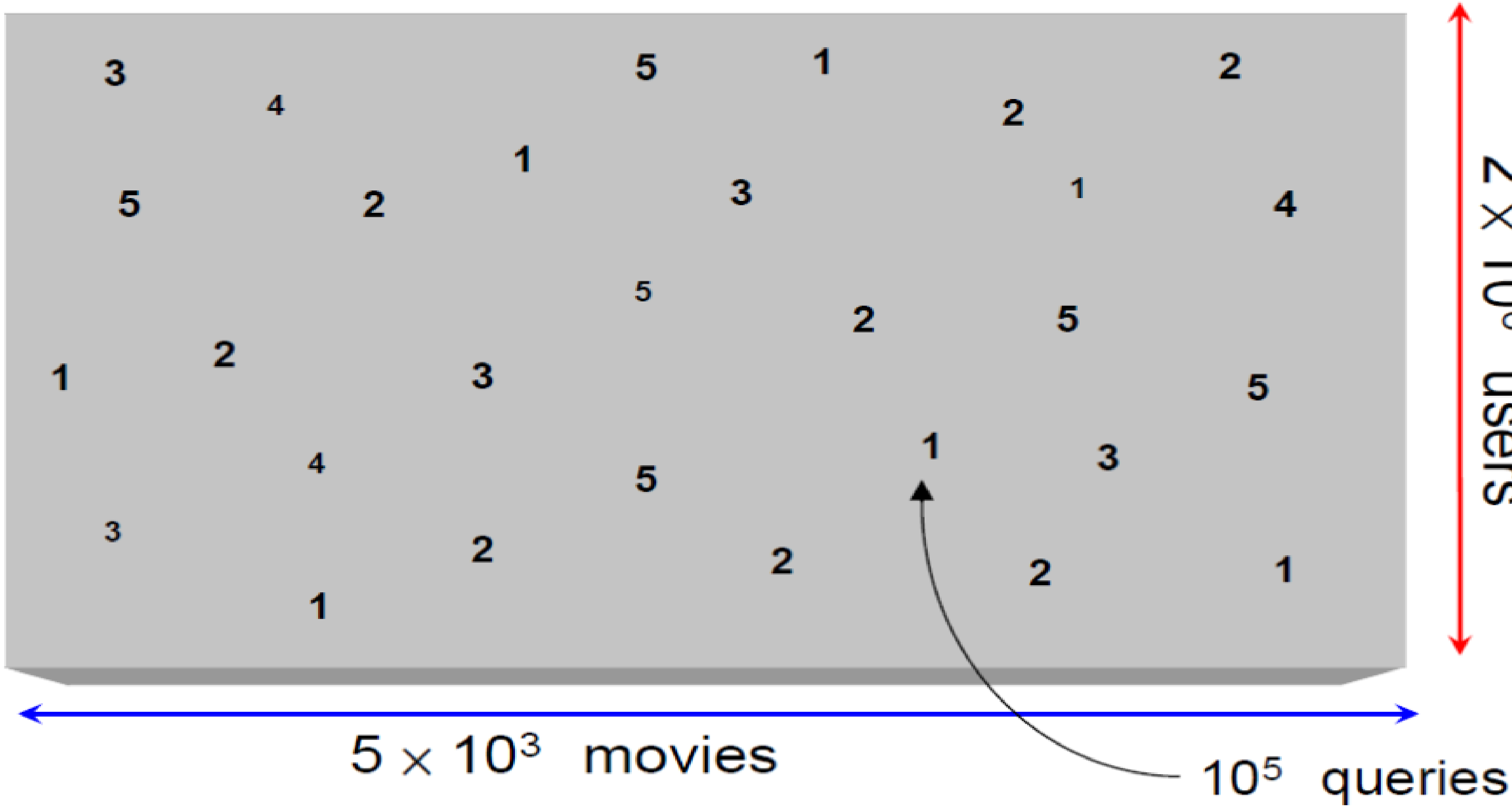
- Netflix為追求最佳collaborative filtering算法的獎，獎金一百萬美金，由Netflix公司提供。
- 比賽方式是所提供的方法要比該公司用戶的方法(Cinematch)好10%. 度量方法是RMSE (root-mean-square-error).

# Netflix問題

- Data format: <user, movie, date of grade, grade>
- Training data: Training set (99,072,112 ratings not including the probe set, 100,480,507 including the probe set)
- Quiz set (1,408,342 ratings), used to calculate leaderboard scores
- Test set (1,408,342 ratings), used to calculate competition scores
- On September 18, 2009, Netflix announced team "BellKor's Pragmatic Chaos" as the prize winner (a Test RMSE of 0.8567)

# A Matrix Completion Approach

- Given less than 1% of movie ratings
- Goal: predict missing ratings



# Matrix Completion

Hypothesis: only small number of independent subclasses.

- Model 1:

$$\min \text{rank}(X) \text{ subject to } X_{ij} = M_{ij} \text{ for } (i,j) \in E$$

  NP-hard

- Model 2:

$$\min \|X\|_* \text{ subject to } X_{ij} = M_{ij} \text{ for } (i,j) \in E.$$

$$\|X\|_* = \sum_i \sigma_i(X) (\text{singular values of } X).$$

  Convex optimization, easy.

# Matrix Completion

- It can be used for recommender systems

- It can also be used for classification

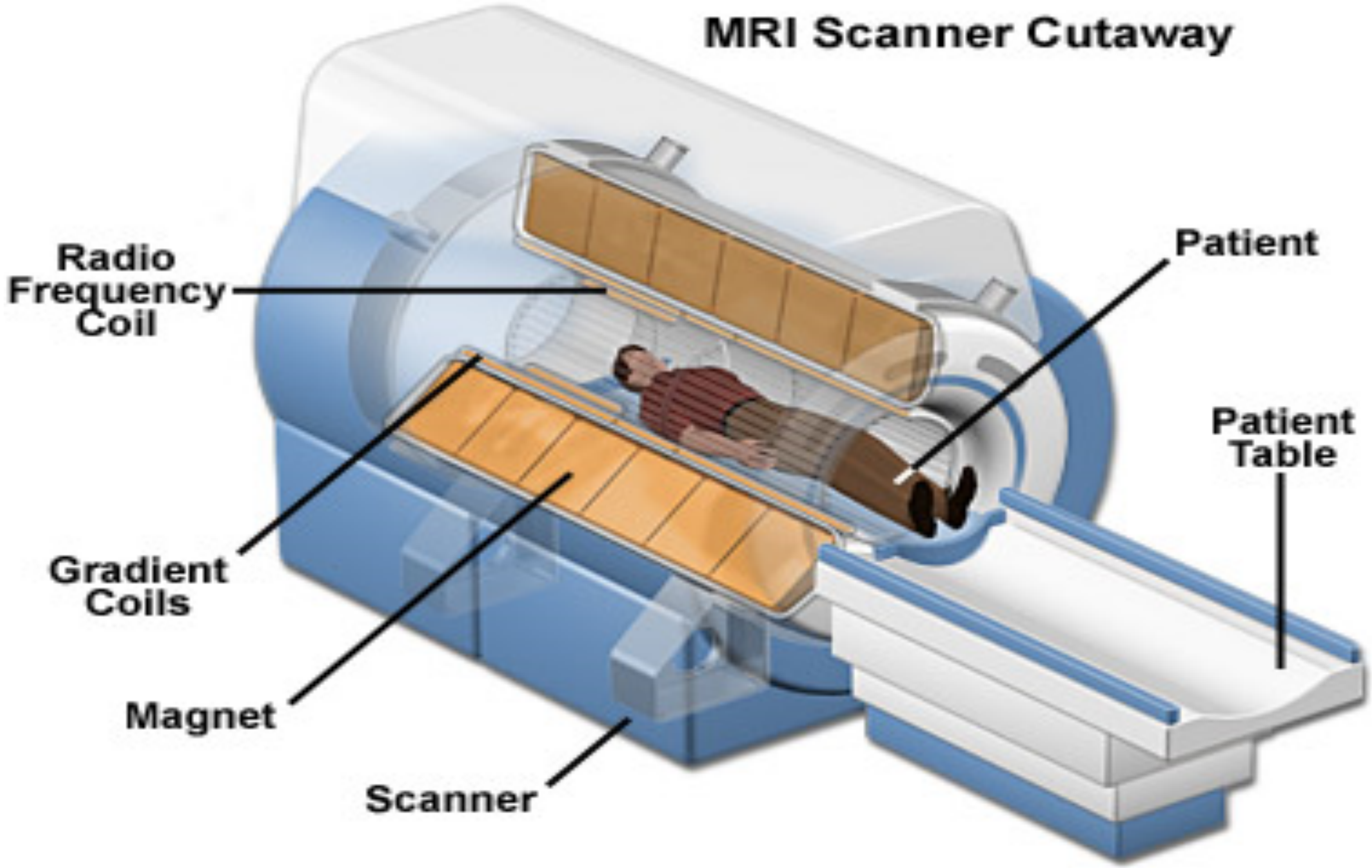
# Part 2 工業界怎麼用數學
# How math is used in Industry?

- How google works
- How Amazon recommends books
- How to reduce imaging time in MRI
- How to denoise a corrupted picture
- How to inpaint a corrupted painting
- Some other info on ``math in industry.''

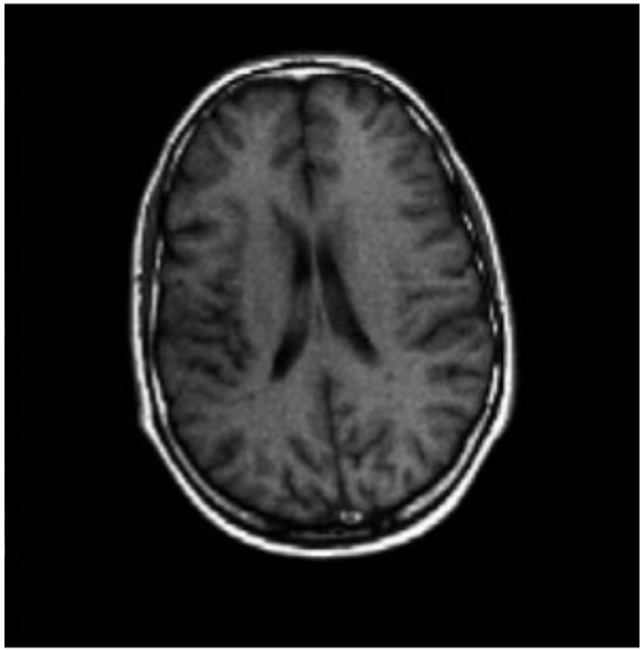
# How to reduce imaging time for MRI

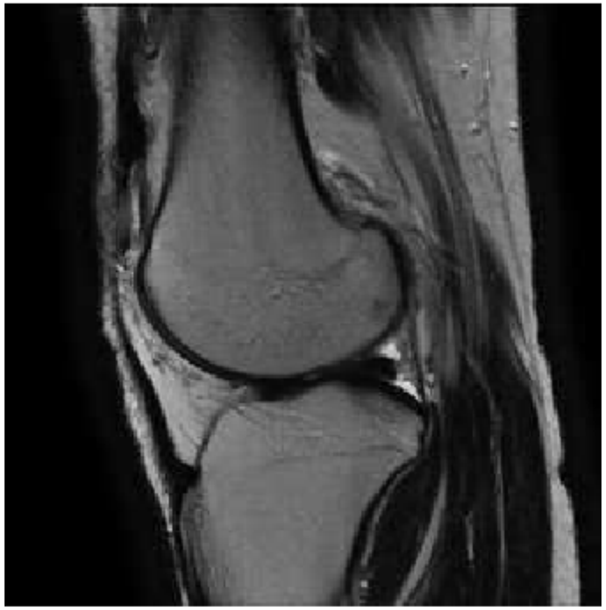
- Motivation from medical imaging
  - Low dose
  - Fast imaging
  - High resolution
- A breakthrough: Compressed Sensing

# Magnetic Resonance Imaging (MRI)



# MRI images





Typical resolution: 256 x 256 or 512 x 512

# MRI history

## The Nobel Prize in Physics 1952

"for their development of new methods for nuclear magnetic precision measurements and discoveries in connection therewith"

**Felix Bloch**

◑ 1/2 of the prize

USA

Stanford University
Stanford, CA, USA

**Edward Mills Purcell**

◑ 1/2 of the prize

USA

Harvard University
Cambridge, MA, USA

## The Nobel Prize in Physiology or Medicine 2003

"for their discoveries concerning magnetic resonance imaging"

**Paul C. Lauterbur**

◑ 1/2 of the prize

USA

University of Illinois
Urbana, IL, USA

**Sir Peter Mansfield**

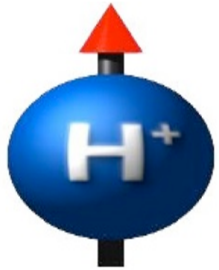
◑ 1/2 of the prize

United Kingdom

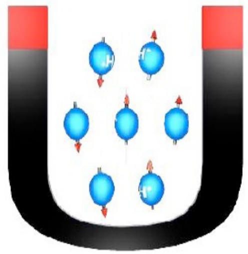
University of Nottingham, School of Physics and Astronomy
Nottingham, United Kingdom

# Basic Principles of Nuclear Magnetic Resonance

- Atoms with odd number of protons and/or neutrons possess nuclear spin angular momentum S
- Associated with S is a magnetic dipole moment



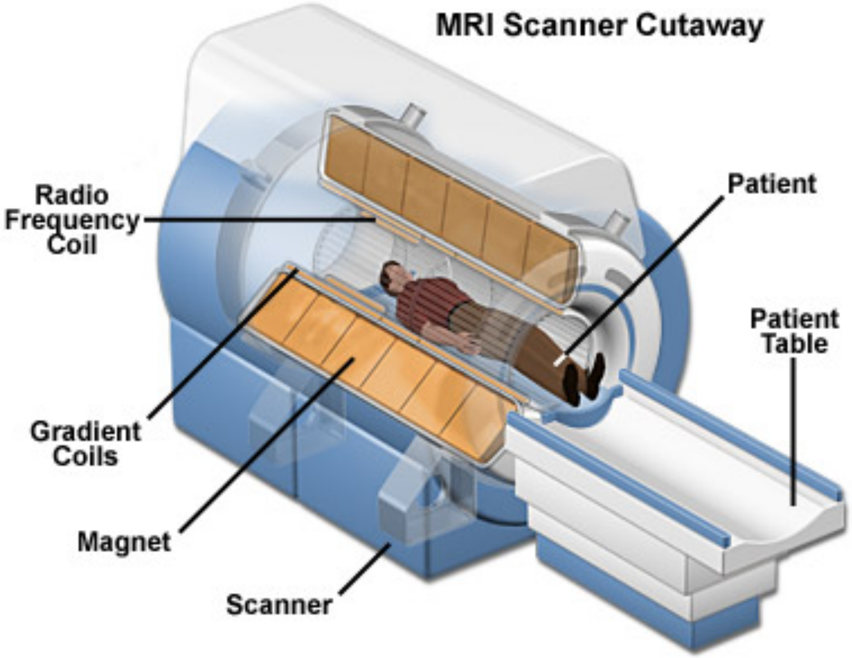

- Magnetic dipole moment rotates under external magnetic field, exhibit magnetic resonance phenomena
- The variation of rotation of spins generate magnetic fluxes and can be recorded

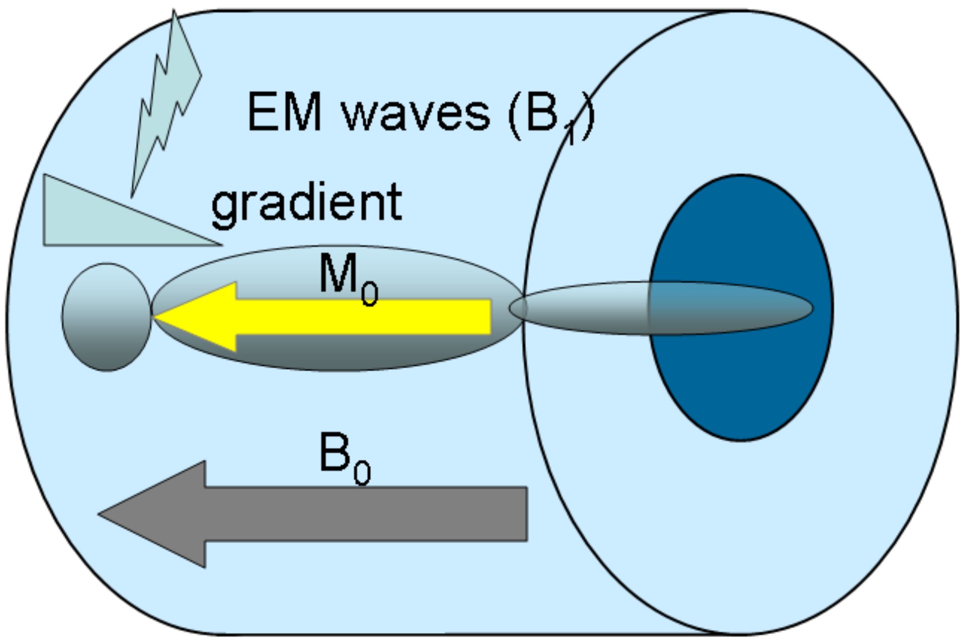


- Hydrogen H+ atoms are abundant in biological specimens

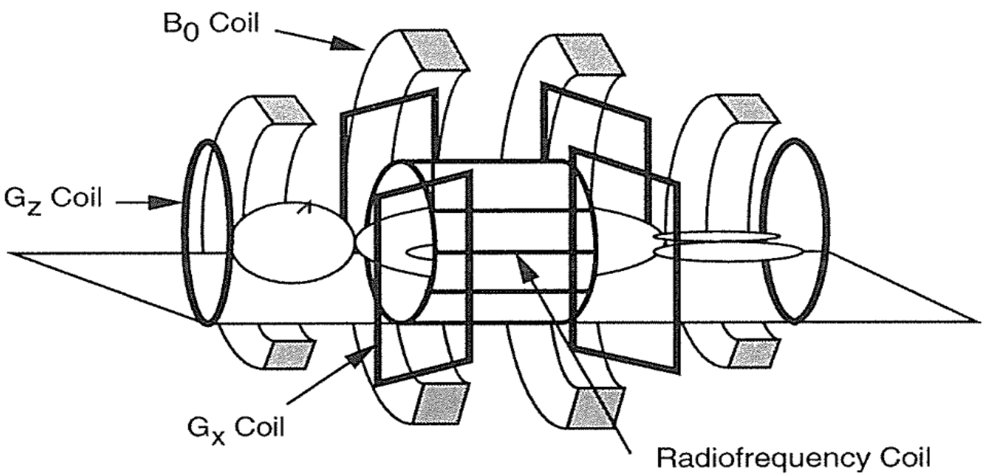
# MRI:
# use magnetic fields to perform

- Relaxation: Main field B0
- Excitation: Radio Frequency (RF) field B1
- Fourier transform: Gradient field G

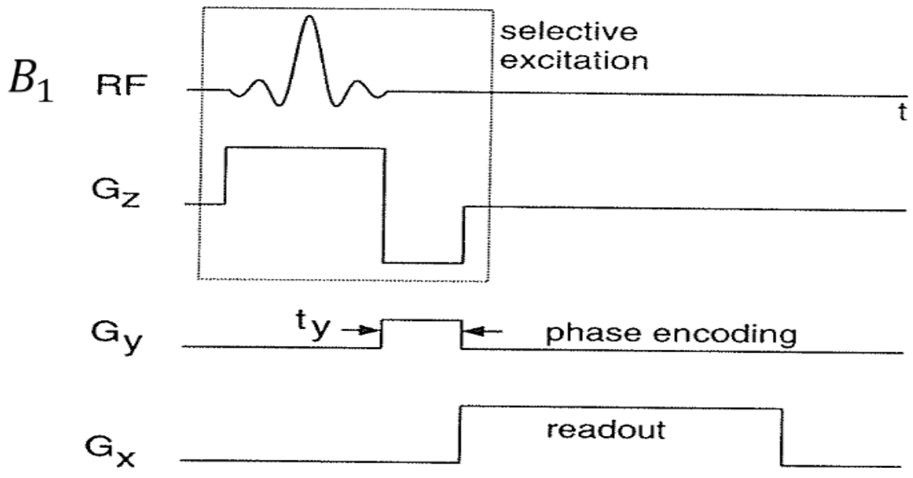


# Magnetic fields in MRI



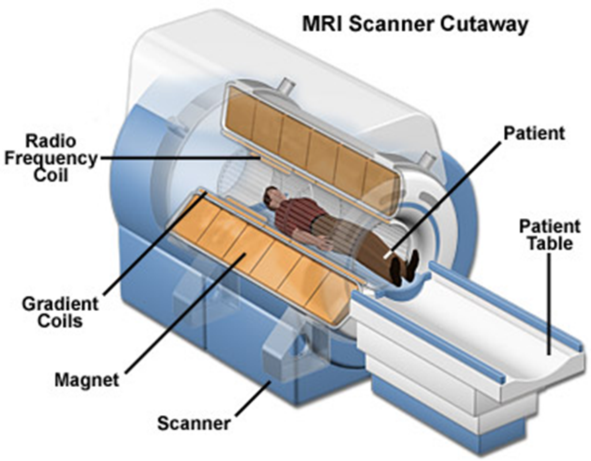

$$B(t) = B_0 + B_1(t) + G(t)$$

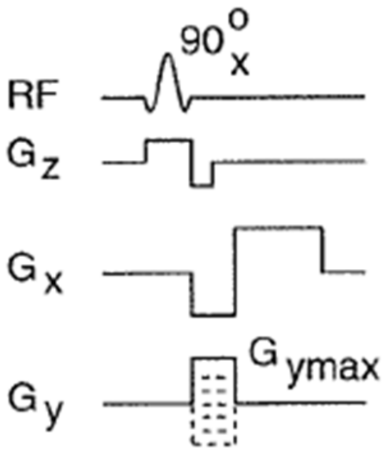


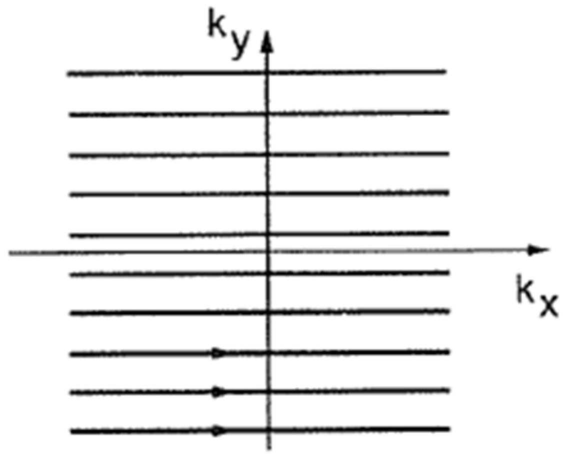
# Principle of MRI

- Relaxation: Main field B0
- Excitation: Radio Frequency (RF) field B1
- Fourier transform: Gradient field G

MRI is a Fourier integrator



# Compressive sensing

- In MRI, we want to reconstruct an $N \times N$ image $f$ from $\hat{f}(\omega)$ defined by

$$\hat{f}(\omega) = \sum_{t \in \mathbb{Z}_N^2} f(t) e^{-2\pi i \omega \cdot t / N}$$

  with $\omega \in \Omega$ with $\#\Omega << N^2$.

- Suppose $N = 512$. We choose $\Omega$ consisting 22 radial lines with 512 uniform sample points on each line.
- The reconstruction can be exact if $f$ is sparse.

E. Candes, J. Romberg, T. Tao 2006
David Donoho 2006

# Compressive sensing

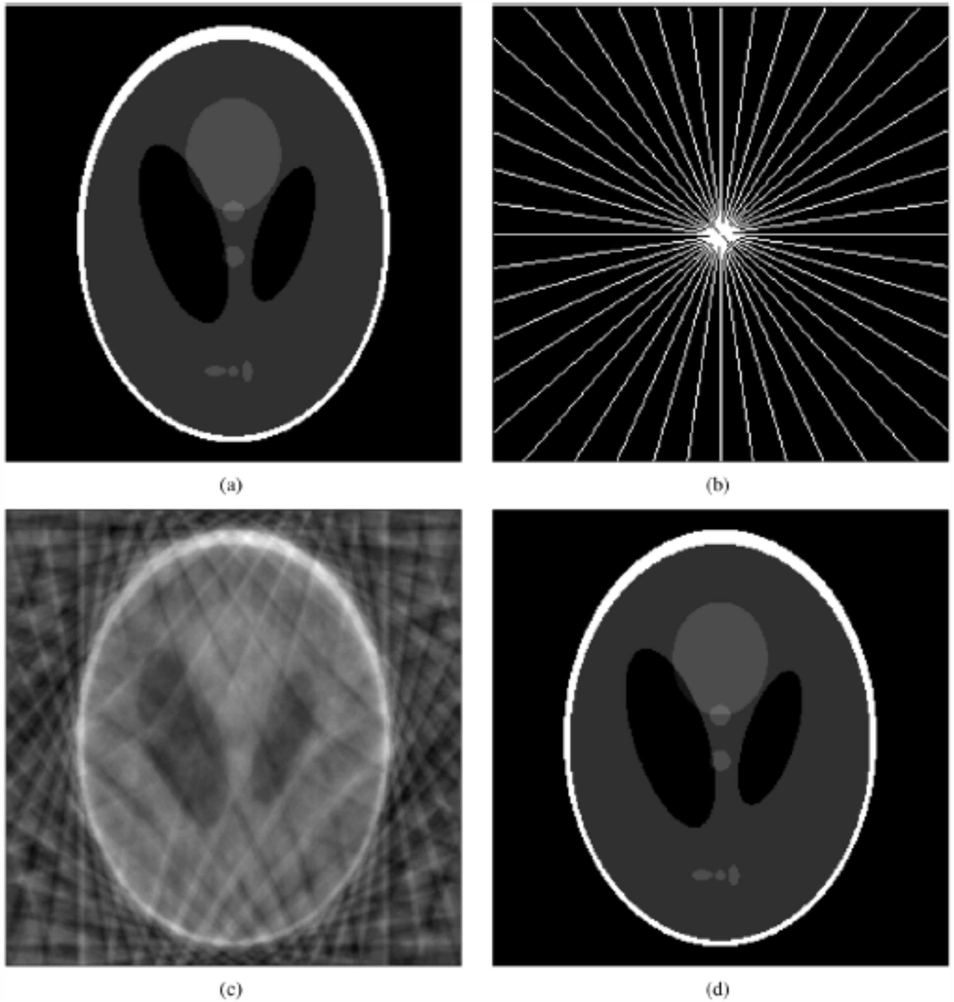



Fig. 1. Example of a simple recovery problem. (a) The Logan–Shepp phantom test image. (b) Sampling domain $\Omega$ in the frequency plane; Fourier coefficients are sampled along 22 approximately radial lines. (c) Minimum energy reconstruction obtained by setting unobserved Fourier coefficients to zero. (d) Reconstruction obtained by minimizing the total variation, as in (1.1). The reconstruction is an exact replica of the image in (a).

Candes,Romberg,Tao 2006

# Compressive sensing in Fourier space

- The result of filtered backprojection is poor if $\Omega$ is sparse.
- The result of TV regularization is exact if $f$ is sparse:

$$\min_u \|\nabla u\|_1 \text{ subject to } \hat{u}(\omega) = \hat{f}(\omega), \omega \in \Omega.$$

# Part 2 工業界怎麼用數學
# How math is used in Industry?

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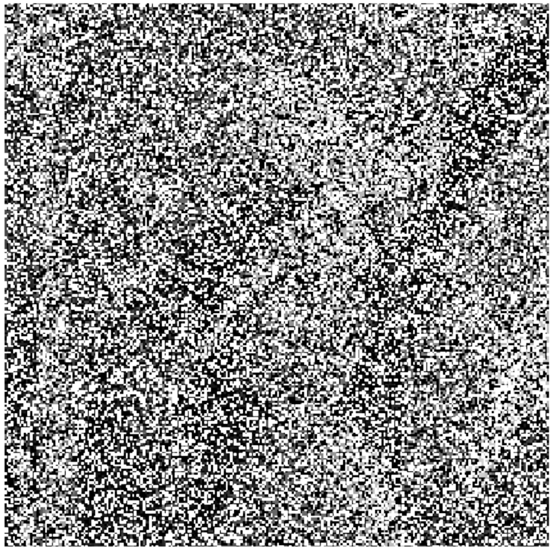
- Some other info on ``math in industry.''

# Denoising



Chan, Ho, Nikolova L1

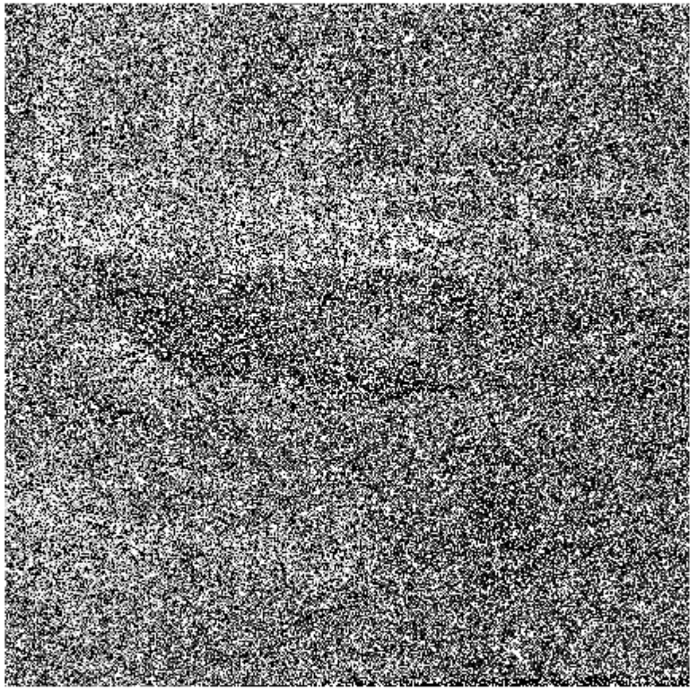
**70% Salt-and-Pepper Noise**

# impulse denoise

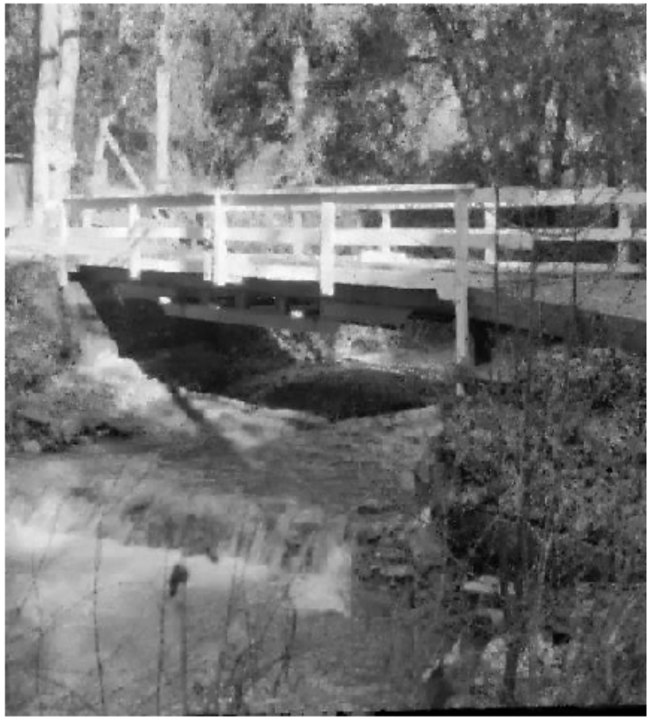
- Impulse noise is Laplace noise. Fidelity function is $\|u - z\|_{L^1}$.
- Edge preserving prior $J(u) = \int |\nabla u|$.
- Denosing by $\min_u F(u)$.

$$F(u) = \beta J(u) + \|u - z\|_{L^1}.$$

# Denoising

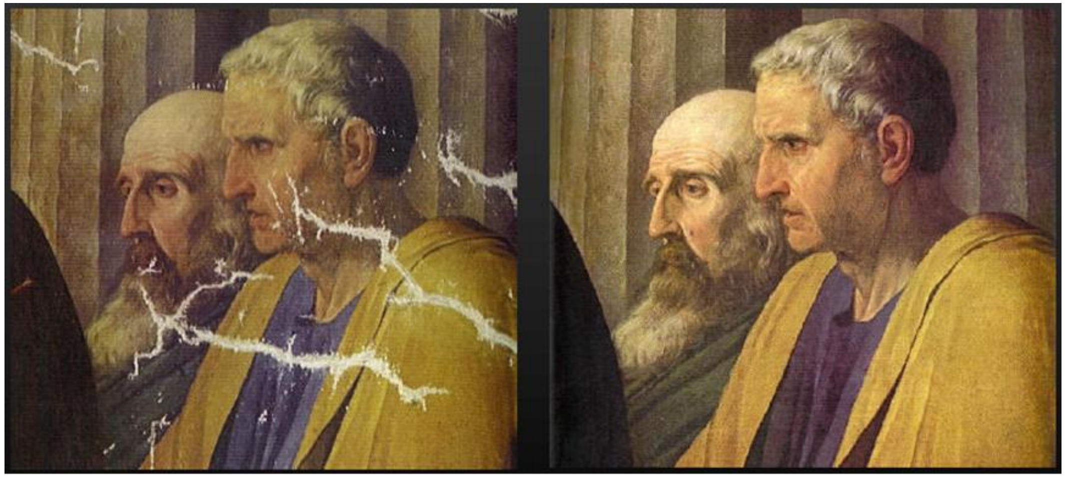


**70% Salt-and-Pepper Noise**



Chan, Ho, Nikolova

# Image Inpainting



***"Image Inpainting : An Overview",***
Guillermo Sapiro


***"Fast Digital Image Inpainting",***
Manuel M. Oliveira,
Brian Bowen,
Richard McKenna
and Yu-Sung Chang

Chiu-Yen Kao

# Image Inpainting

- $z$: observed image, $D$ missing inpainting domain
- Energy functional:

$$E(u, D) = \gamma E_p + \int_{\Omega \setminus D} |u - z|^2$$

- Prior functional:

TV: $E_p = \int_\Omega |\nabla u|$

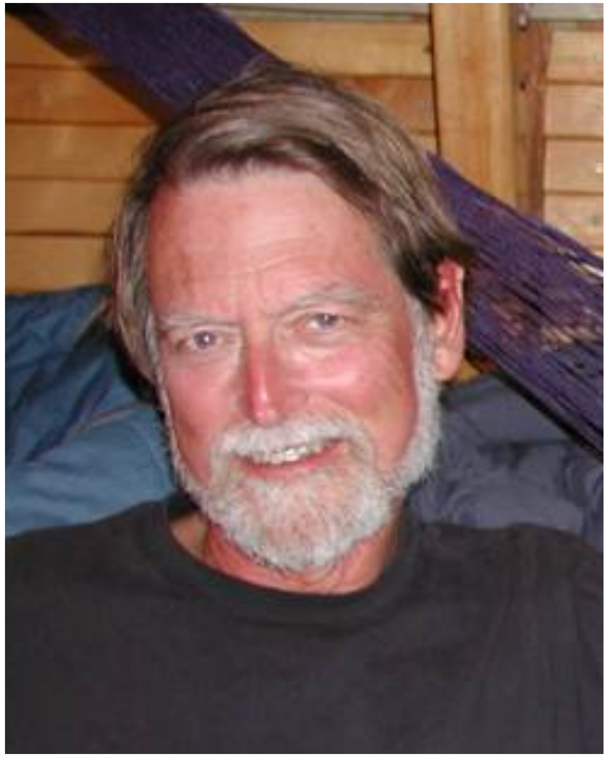
Elastica: $E_p = \int_\Omega \phi(\kappa) |\nabla u|$

where

$$\phi(s) = \alpha + \beta s^2, \ \kappa = \nabla \cdot \left[ \frac{\nabla u}{|\nabla u|} \right].$$

Stanley Osher

Tony F Chan



David Mumford

Some Applied Mathematicians in Image Processing

# Compressive Sensing (2004)

David Donoho

Emmanual Candes

Justin Romberg

Terence Chi-Shen Tao

Candes, Romberg, Tao, Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information, IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. 52, NO. 2, FEBRUARY 2006 cited 4894

DL Donoho, Compressed sensing, Information Theory, IEEE Transactions on, 2006 , cited 6716

# 關於工業中數學的一些資料

- 85 Things Mathematicians Have Been Hired to Do In Industry (American Math Society)

- European Consortium for Mathematics in Industry (ECMI) :
  Progress in Industrial Mathematics ECMI2006-2014

- Journal of Mathematics in Industry

# Part 3 How to prepare
# 如何做好準備

- 學會寫程式
- 傳統數學在新時代要有新思維(建模與應用)
- 多接觸應用課題，從實作當中學習

# 歐巴馬鼓勵年輕人要會程式

- 2013年12月8日歐巴馬在計算機科學周演講，鼓勵美國年輕人要會寫程式:
  - “If we want America to stay on the cutting edge, we need young Americans like you to master the tools and technology that will change the way we do just about everything.”
  - ’’Don’t just buy a new video game, make one.’’
  - ’’Don’t just download the latest app, help design it.’’
  - “Don’t just play on your phone, program it.”
- 編寫程式是未來社會的基本技能，別成了程式文盲。

# 新時代 新思維　新數學

- 傳統數學在新時代要有新思維
- 新的應用課題賦予傳統數學新的生命
  - 線性代數中的Singular Value Decomposition在數據分析、影像處理十分重要
  - 各種大尺度數值算法十分重要
  - 優化、數學規劃等都有新的應用
  - 圖論在社群網路研究有新的應用
  - 微分方程、偏微分方程、變分學、反問題等仍是連續世界建模的基本工具
  - 機率、統計、不確定性的量化分析等也是建模的工具

# 做中學

- 多接觸應用問題
- 從實作中學
- 從簡單的東西做起
- 學習將複雜的東西化成簡單的東西

# 一個小故事

- 一個數學系畢業生組織的讀書會
  - Taiwan R User group and Data Mining
  - http://www.meetup.com/Taiwan-R/
  - Python, Git
- R 是目前最紅的 Open Source 統計語言，而且，不只是對於傳統的統計檢定來說，或是，對於新世代的 Machine Learning 和 Data Mining 的許多技術而言，R 都有很完整的套件支援。因此，可以很快速的在R上面運用各種 Machine Learning 的技術來分析資料。
- 他們開了一家Data Mining 公司。
- 大量數據分析: http://www.revolutionanalytics.com/

謝謝大家