5th homework
Due date: 10/21

Let I_n be the $n \times n$ identity matrix and 0_n be the n$\times n$ zero matrix. There are **seven** problems.

Exercise 1. Evaluate the following determinants

$$(1)\det\begin{pmatrix} 1 & -2 & 3 & -12 \\ -5 & 12 & -14 & 19 \\ -9 & 22 & -20 & 31 \\ -4 & 9 & -14 & 15 \end{pmatrix}; \quad (2)\det\begin{pmatrix} 1 & 1 & -1 & -1 \\ i & -i & i & -i \\ 1 & 1 & -1 & -1 \\ i & -i & i & -i \end{pmatrix} \qquad i = \sqrt{-1}.$$

Exercise 2. Let

$$A = \begin{pmatrix} 2 & 6 & 4 \\ 3 & 3 & 1 \\ 5 & 8 & 4 \end{pmatrix}.$$

Find the adjoint matrix $\mathrm{adj}(A)$ and the inverse A^{-1}.

Exercise 3. Let x_1, x_2, x_3 be three real numbers such that

$$x_1 + x_2 + x_3 \neq 0 \text{ and } x_i \neq x_j \text{ for } i \neq j.$$

Show that the set $\{(x_1, x_2, x_3),\ (x_3, x_1, x_2),\ (x_2, x_3, x_1)\}$ is a basis of $\mathbf{R}^3$.

Exercise 4. Find $a \in \mathbf{R}$ such that vectors

$$(1, 3, 1),\ (a, 4, 3),\ (0, 1, 1) \in \mathbf{R}^3$$

are linearly dependent.

Exercise 5. Let $A, B \in M_n(F)$. Show that

$$\det\begin{pmatrix} A & B \\ B & A \end{pmatrix} = \det(A + B) \cdot \det(A - B).$$

(You may use the formula $\det\begin{pmatrix} A & 0_n \\ C & D \end{pmatrix} = \det A \cdot \det D$).

Exercise 6. Prove that

$$\det\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & z^2 & y^2 \\ 1 & z^2 & 0 & x^2 \\ 1 & y^2 & x^2 & 0 \end{pmatrix} = (x + y + z)(x + y - z)(x - y + z)(x - y - z).$$

Exercise 7. If A, B and C are three interior angles of a triangle ΔABC, prove that

$$\det \begin{pmatrix} \cos 2A & \frac{\cos A}{\sin A} & 1 \\ \cos 2B & \frac{\cos B}{\sin B} & 1 \\ \cos 2C & \frac{\cos C}{\sin C} & 1 \end{pmatrix} = 0$$