9th homework
Due date: 5/19

Exercise 1. Let

$$A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & -2 & 1 \\ 1 & 3 & -2 \end{pmatrix} \in \mathrm{M}_3(\mathbf{R}).$$

Let $b = (3, -2, 1)^{\mathrm{t}} \in \mathbf{R}^3$. Find the minimum

$$\min_{x \in \mathbf{R}^3} \|Ax - b\|.$$

For any matrix $A \in \mathrm{M}_n(\mathbf{R})$, denote by ρ_A the spectral radius of A.

Exercise 2. Let $A \in \mathrm{M}_n(\mathbf{R})$ be an irreducible matrix. Suppose that $u \in (\mathbf{R}_{\geq 0})^n$ with

$$Au - \rho_A u \in (\mathbf{R}_{\geq 0})^n.$$

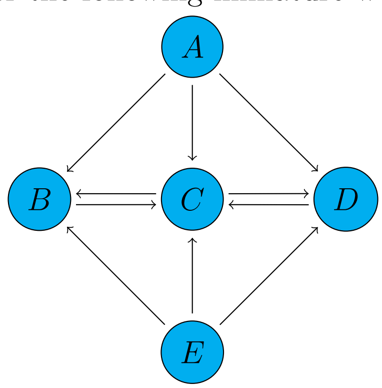
Prove that $Au = \rho_A u$.

(Hint: consider $w = (A + I_n)^{n-1} u$ and show that $Aw - \rho_A w \in (\mathbf{R}_{>0})^n$.

Exercise 3. Let $A \in \mathrm{M}_n(\mathbf{R})$ be an irreducible matrix. Show that if $u \in (\mathbf{R}_{\geq 0})^n$ is an eigenvector of A, then $u \in (\mathbf{R}_{>0})^n$.

Exercise 4. Let $A = (a_{ij}) \in \mathrm{M}_n(\mathbf{R})$ be an irreducible matrix such that $a_{11} > 0$. Show that A is regular.

Exercise 5. Consider the following miniature web:



(1) Take the damping factor $\alpha = 0.9$. Find the Google matrix

$$G = \alpha P + \frac{1 - \alpha}{5} \cdot E \in \mathrm{M}_5(\mathbf{R})$$

associated with the above web, where P is the transition matrix of the web and $E = e^{\mathrm{t}} e$ with $e = (1, 1, 1, 1, 1)$.

(2) Compute the Perron vector of G. You may need to use matrix calculator for this problem.