2nd homework
Due date: 9/30

Let F be either $\mathbf{Q}$, $\mathbf{R}$, or $\mathbf{C}$.

Exercise 1. Show that $(1,1,0,1)$, $(1,0,1,1)$, $(0,0,1,1)$ and $(0,1,0,2)$ form a basis of F^4.

Exercise 2. Let $S = \{u_1, u_2, u_3\} \subset F^4$ be a subset of vectors in F^4 given by

$$u_1 = (1, 10, -6, -2), \quad u_2 = (-2, 0, 6, 1), \quad u_3 = (3, -1, 2, 4).$$

Let $W := \mathrm{span}_F S$. Find $\dim_F W$.

Exercise 3. Let V be a vector space over F and let $\{v_1, v_2, v_3\}$ be a basis of V. Show that

$$\{v_1 + v_2 + v_3, v_1 + 2v_2 + 4v_3, v_1 + 3v_2 + 9v_3\}$$

is also a basis of V.

Exercise 4 (Extension lemma)**.** Let V be a finite dimensional vector space over F with $\dim_F V = n$. Let $v_1, v_2, \ldots, v_k \in V$ be linearly independent vectors. Show that there exists $(n-k)$ vectors $v_{k+1}, v_{k+2}, \ldots, v_n$ such that $\{v_1, v_2, \ldots, v_n\}$ is a basis of V.

Exercise 5. Let V be a finite dimensional vector space over F. Let W_1 and W_2 be subspaces of V. Define

$$W_1 + W_2 := \{x + y \in V \mid x \in W_1, y \in W_2\}.$$

(1) Show that $W_1 + W_2$ is a subspace of V.
(2) Use the previous exercise *Extension lemma* to show

$$\dim_F(W_1 + W_2) + \dim_F(W_1 \cap W_2) = \dim_F W_1 + \dim_F W_2.$$