3rd homework
Due date: 3/24

Exercise 1. Let $v_1 = (3, 0, 4)$, $v_2 = (-1, 0, 7)$ and $v_3 = (2, 9, 11)$ be vectors in $\mathbf{R}^3$ equipped with the standard inner product $\langle (a_1, a_2, a_3), (b_1, b_2, b_3) \rangle = a_1b_1 + a_2b_2 + a_3b_3$. Let W be the subspace spanned by v_2 and v_3. Find $\mathrm{Proj}_W(v_1) =?$.

Exercise 2. If $(V, \langle\ ,\ \rangle)$ is a finite dimensional inner product space over $\mathbf{C}$, show that any linear functional $\ell : V \to \mathbf{C}$ is bounded.

Exercise 3. Let $(V, \langle\ ,\ \rangle)$ be a Hilbert space.

(1) If S is any subset of V, show that
$$S^\perp := \{x \in V \mid \langle x, s \rangle = 0 \text{ for all } s \in S\}$$
is a closed subspace of V.

(2) Show that if W is a subspace of V, then W is closed if and only if $W = W^{\perp\perp}$.

Exercise 4. Let V be the space $\mathbf{C}^2$ with the standard inner product. Let $T : \mathbf{C}^2 \to \mathbf{C}^2$ be the linear transformation defined by $T(1, 0) = (1, -2)$ and $T(0, 1) = (i, -1)$. Let T^* be the adjoint of T. Find $T^*(x_1, x_2)$ for any $(x_1, x_2) \in \mathbf{C}^2$.

Exercise 5. Let V be the space of all real valued continuous functions on $[0, 1]$ with the inner product
$$\langle f, g \rangle := \int_0^1 f(t)g(t)dt.$$
Define $T : V \to V$ by
$$T(f)(x) := \int_0^x f(t)dt, \quad x \in [0, 1].$$
Show that the adjoint of T exists.