# 11th homework
# Due date: 12/09

**Exercise 1** (30pts)**.** Determine the Jordan form $J$ of the following matrices $A$:

(1)
$$A = \begin{pmatrix} -1 & -1 & 4 & 0 \\ 1 & -3 & 7 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & -1 & 1 & 0 \end{pmatrix}.$$

(2)
$$A = \begin{pmatrix} 5 & -5 & 2 & 4 \\ 2 & -1 & 2 & -2 \\ 1 & -1 & 4 & 1 \\ -1 & 2 & -1 & -4 \end{pmatrix}.$$

(3)
$$A = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ -3 & 2 & 1 & 1 \\ 3 & -6 & 1 & 4 \end{pmatrix}.$$

Find $P \in M_4(\mathbf{Q})$ such that $P^{-1}AP = J$.

**Exercise 2.** Supose that

(1) $A \in M_8(\mathbf{R})$ is a real matrix with the minimal polynomial
$$m_A(x) = x^2(x-2)^3;$$

(2) $B$ is a real matrix with the characteristic polynomial
$$\mathrm{ch}_B(x) = (x+2)^4(x-5)^2.$$

Find all possible Jordan forms of $A$ and $B$.

**Exercise 3.** Give an example of two matrices $A, B \in M_n(\mathbf{C})$ such that $\mathrm{ch}_A(x) = \mathrm{ch}_B(x)$ and $m_A(x) = m_B(x)$, but $A$ and $B$ have different Jordan forms.

**Exercise 4.** Let $A, B \in M_n(\mathbf{C})$. Suppose that
$$\begin{pmatrix} A & 0_n \\ 0_n & A \end{pmatrix} \text{ and } \begin{pmatrix} B & 0_n \\ 0_n & B \end{pmatrix} \in M_{2n}(\mathbf{C})$$
are similar. Show that $A$ and $B$ are similar.