10th homework
Due date: 12/02

Exercise 1. Let $A, B \in M_n(F)$. Suppose that the characteristic polynomial of A is given by

$$\mathrm{ch}_A(x) = (x - \lambda_1)^{n_1} \cdot \cdots \cdot (x - \lambda_s)^{n_s}$$

with distinct roots λ_i. Assume that A and B have the same characteristic polynomial and the same minimal polynomial and that $n_i \leq 3$ for $1 \leq i \leq s$. Show that A is similar to B.

Exercise 2. Let $T : V \to V$ be a linear transformation with the charateristic polynomial $\mathrm{ch}_T(x) = x^4 + 3x^2 + x - 2$. If V is T-cyclic, show that there exists a basis $\mathscr{A}$ of V such that

$$[T]_{\mathcal{A}} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -3 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 2 & 0 & 0 & 0 \end{pmatrix}$$

Exercise 3. For $\alpha \in F$, define

$$A := \begin{pmatrix} 0 & 1 & \alpha & 0 \\ -1 & 2 & 0 & \alpha \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 2 \end{pmatrix}.$$

Let J be the Jordan form of A. Determine J in terms of α and find P such that $P^{-1}AP = J$.

Exercise 4. Let

$$A = \begin{pmatrix} 4 & -4 & -11 & 11 \\ 3 & -12 & -42 & 42 \\ -2 & 12 & 37 & -34 \\ -1 & 7 & 20 & -17 \end{pmatrix}.$$

(1) Show that $\mathrm{ch}_A(x) = (x - 3)^4$.
(2) Determine the Jordan form J of A.
(3) Find an invertible $P \in M_4(\mathbf{Q})$ such that $P^{-1}AP = J$.