

18.

Hint: Convert the differential equation into the standard form (as the same as equation 1 in textbook), and then find the integrating factor to finish the integration.

20.

Hint: Let  $u = y - 1$

$\Rightarrow \frac{du}{dx} = \frac{dy}{dx}$  ,  $u(0) = y(0) - 1 = 1$  , and then we have a new differential equation:

$$\frac{du}{dx} + \left(\frac{3x}{x^2 + 1}\right)u = 0, u(0) = 1$$

Solve  $u(x)$  and then get  $y(x)=u(x)+1$ .

22.

Hint: Just find the integrating factor and then integrate it directly. The constant  $C$  will appear after integration.

24.

Hint: Let  $u = y^{-1}$  and use the method in Exercise 23 to solve the differential equation of  $u(x)$ . And then we get  $y(x) = [u(x)]^{-1} = \frac{1}{u(x)}$  .

26.

Hint: Convert the equation into a differential equation of  $u(x)$  , where

$$u(x) = \frac{dy}{dx}.$$

Solve  $u(x)$  first and then  $y(x) = \int u(t)dt$  can be solved.

28.

Hint: Use equation 4 in textbook and then solve the differential equation.

You may use the technique of integration by parts to solve the integration.

30.

Hint: Use the results in Exercise 29 and then solve  $Q(t)$  . The current

$$I(t) = \frac{d}{dt}Q(t).$$

32. Solve the equation in Exercise 31 to get the solution:

$$P(t) = M + Ce^{-kt}, k > 0$$

For  $P(0) = 0$  ,  $P(t) = M[1 - e^{-kt}]$  .

Now, try to find the value of  $M_1$  and  $M_2$  respectively by the given values.

34.

Hint: Let  $y(t)$  be the amount of chlorine in the tank at time  $t$  . What is the

amount of liquid in the tank at time  $t$  ? The concentration of chlorine at time  $t$  ? The rate of chlorine leaving the tank at time  $t$  ?

Find out the differential equation:  $\frac{dy}{dt} = p(t)$  and then solve  $y(t)$  .

36.

Hint: Use the equation in Exercise 35(a) :

$$v = \frac{mg}{c} (1 - e^{\frac{-ct}{m}})$$

Differentiate it with respect to  $m$  to find  $\frac{dv}{dm}$ .

Can you ensure that  $\frac{dv}{dm} > 0$  for all  $m > 0$  ?

(You may use the substitution  $u = \frac{-ct}{m}$  to simplify the expression of  $\frac{dv}{dm}$  , where  $u > 0$  for all  $m > 0$  and all  $t > 0$  .)